Math 223 Midterm 1 Review

1. Do the three planes \(2x_1 - 3x_2 - x_3 = -1\), \(x_1 - x_2 + x_3 = -4\), and \(5x_1 + 4x_2 - x_3 = 1\) have at least one common point of intersection? Explain. If they have a single point of intersection, find it. If there are multiple points, give a way of generating them all.

2. Construct a \(3 \times 3\) nonzero matrix, \(A\), such that the vector \(\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}\) is a solution to the equation \(Ax = 0\). Explain your reasoning.

3. Let \(v_1 = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}\), \(v_2 = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}\), and \(\vec{u} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}\). Is \(\vec{u} \in \text{Span}\{\vec{v}_1, \vec{v}_2\}\)? Explain your reasoning.

4. Consider the following \textbf{unbalanced} chemical equation for producing ammonia (NH\(_3\)). Use the technique of vector equations to balance the chemical reaction.

\[
\text{CaCN}_2 + \text{H}_2\text{O} \rightarrow \text{NH}_3 + \text{CaCO}_3
\]

5. Determine the value(s) of \(k\) such that the set of vectors \(\left\{ \begin{bmatrix} 1 \\ k \end{bmatrix}, \begin{bmatrix} k \\ k+2 \end{bmatrix} \right\}\) is linearly independent.

6. Let \(T: \mathbb{R}^2 \rightarrow \mathbb{R}^2\) be a linear transformation with standard matrix \(A = [\vec{a}_1 \vec{a}_2]\), where \(\vec{a}_1\) and \(\vec{a}_2\) are shown below. Using the figure, draw the image of the vector \(\begin{bmatrix} -1 \\ 3 \end{bmatrix}\) under the transformation, \(T\). Explain your reasoning and sketch intermediate operations used to find the image.

7. Find a matrix for a linear transformation that rotates vectors in \(\mathbb{R}^2\) by an angle of \(\frac{\pi}{3}\) counter clockwise.
8. Find a $3 \times 3$ transformation matrix that takes all vectors in $\mathbb{R}^3$ and projects them onto the $x_1x_3$ plane.

9. Consider the transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by $T(\vec{x}) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \cdot \vec{x}$. Show that $T$ is a linear transformation.