Math 223 Midterm 2 Review

1. Suppose the columns of a $5 \times 5$ matrix, $A$, are linearly independent, what can you say about the solutions to the equation $A \bar{x} = \bar{b}$? Explain your reasoning.

2. Suppose in reducing the matrix, $A$, you get
\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] What can you say about the columns of $A$ relative to $\mathbb{R}^4$?

3. Suppose the columns of the $n \times n$ matrix, $A$, are linearly independent. What can you say about the columns of the matrix $A^3$ relative to $\mathbb{R}^n$?

4. Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 7 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 3 \\ -1 \\ 5 \\ 2 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 2 \\ 0 \\ -3 \\ 4 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 3 \\ 3 \\ 6 \\ 12 \end{bmatrix}$. Determine if $\vec{w}$ is in the subspace generated by $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

5. Consider the matrix $A = \begin{bmatrix} 3 & -6 & 9 & 0 \\ 2 & -4 & 7 & 2 \\ 3 & -6 & 6 & -6 \end{bmatrix}$. Find a basis for $\text{Col} \, A$ and a basis for $\text{Nul} \, A$.

6. Suppose the columns of an $m \times n$ matrix, $A$, form a basis for $\mathbb{R}^m$. What can you say about $n$?

7. Consider the following sketch. Let $\vec{w}$ be a vector in a subspace $H$ with basis $B = \{\vec{v}, \vec{u}\}$ as shown. Find the $B$-coordinate vector of $\vec{w}$.
8. Let \( \vec{x} \) be a vector in a subspace \( H = \text{Span}\{\vec{v}_1, \vec{v}_2\} \) with basis \( B = \{\vec{v}_1, \vec{v}_2\} \). Show that \( \vec{x} \in H \) and find \([\vec{x}]_B\) when \( \vec{v}_1 = \begin{bmatrix} 15 \\ -5 \\ 12 \\ 7 \end{bmatrix} \), \( \vec{v}_2 = \begin{bmatrix} 14 \\ -10 \\ 13 \\ 17 \end{bmatrix} \), and \( \vec{x} = \begin{bmatrix} 16 \\ 0 \\ 11 \\ -3 \end{bmatrix} \).

9. Consider the pentagon shown below on the left with an area of 26.5 square units. If the vertices of the pentagon are transformed by the transformation matrix \( A = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix} \) as shown on the right, what is the area of the pentagon on the right?