Lab 9: Basic Integration

Introduction

The purpose of this laboratory experience is to develop fundamental methods for finding antiderivatives so that they can be used along with the Fundamental Theorem of Calculus to find definite integrals. With this in mind, we will work from specific cases to try and explain why some antiderivatives can be found easily and others cannot.

Laboratory Experience

From the Fundamental Theorem of Calculus, we know that if we want to calculate $\int_a^b f(x)\,dx$, we simply have to find an antiderivative F of the function f and find the difference $\int_a^b f(x)\,dx = F(b) - F(a)$. This works well for functions for which we can easily find the antiderivative. You may recall functions such as $f(x) = 3x^2$ allow us to easily see the antiderivative, x^3, by inspection, since it is a simple application of the power rule. On the other hand, if $f(x) = 5x\sin(x^2)$, seeing the antiderivative is not so easy. In this lab we will explore some specific structures of functions that let us more easily see where the derivative came from so that we can go backward to find the antiderivative.

1. Consider the following functions. Use your CAS to try and find the antiderivatives.
   a. $\int \cos(x^2)\,dx$

   b. $\int x^2\cos(x^2)\,dx$

   c. $\int x\cos(x^2)\,dx$

   d. $\int 2x\cos(x^2)\,dx$

   e. For the attempted antiderivatives in parts 1(a)-1(d), which could the CAS find and why do you think it could for those and not the others?

2. In this question, we will use what you found in question 1 to try and “fix” and integral by creating an integral that still uses the “main” function (for example in parts (a) and (b), e^{x^2}), but also contains a “partner” function so that the antiderivative can be found. Note that here we are not necessarily going to be finding the antiderivative of the original function given, but rather a slightly different function instead where the antiderivative can more easily been seen. Consider the following antiderivatives, explain how you might change the function so that the CAS will be able to antidifferentiate it and then test your edits to see if the CAS can do it. Explain your reasoning for the change in the function.

   a. $\int e^{x^2} dx$

   b. $\int xe^{x^3} dx$

   c. $\int \sin(\tan(x)) dx$

   d. $\int \sqrt{1+x^4}\, dx$

3. Now that you have edited functions to make them easier to antidifferentiate, you will now create two functions for which you can easily see the antiderivative.

    a. Devise two functions that can be easily antidifferentiated. The only restriction is that each of your functions must be the product of two other functions.

    b. Given the patterns you have seen in this lab, consider the integral $\int \square e^{\triangle} dx$. Explain how $\square$ and $\triangle$ must be related in order to easily find the antiderivative.

    c. Without using CAS, find the antiderivative $\int x^2 \sec^2\left(x^3\right) dx$. Check your result using the CAS. Explain your strategy.

    d. What rule of differentiation are you undoing using your strategy?