Lab 13: Hydrostatic Force—Dam It

Activity Overview:
Students will use pressure probes to model the hydrostatic force on a dam and calculate the total force exerted on it.

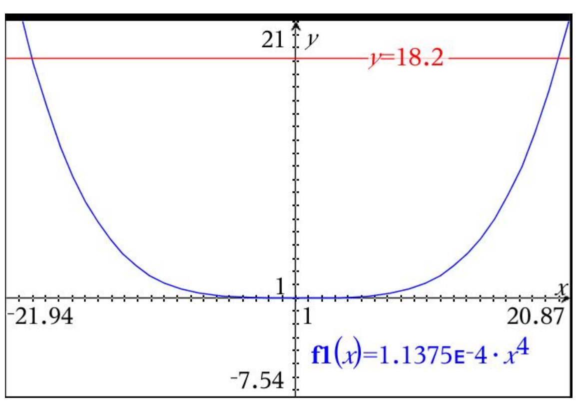
Materials

- *TI-Nspire™ CAS handheld*
- Vernier Gas Pressure Sensor
- 1.5 m PVC pipe with capped off end
- 1.5 m clear plastic aquarium tubing
- fishing sinkers heavy enough to pull the tubing to the bottom of the pipe
- paperclip to attach sinkers to aquarium tubing
- meter stick or tape measure

Problem Statement

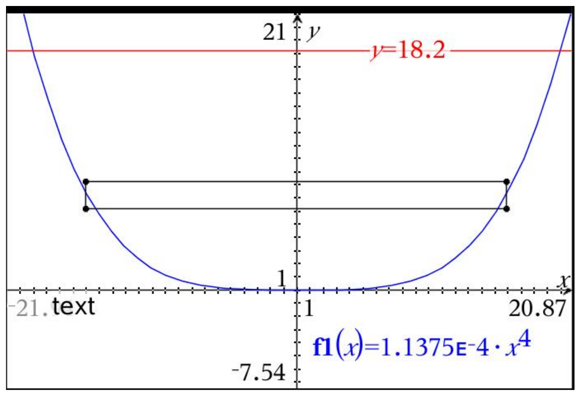
In this activity, you will use a gas pressure sensor to model pressures at varying depths in water. Using these data, you will create a mathematical model for the force of water on a dam at various depths and then use integration to calculate the total force exerted on the dam by the water.

Consider a river whose cross sectional area at the point where a dam is to be constructed is modeled by the function $f(x) = 0.00011375x^4$. This river is 40 meters across at this point and the depth at center of the river is 18.2 meters.



Finding a Model

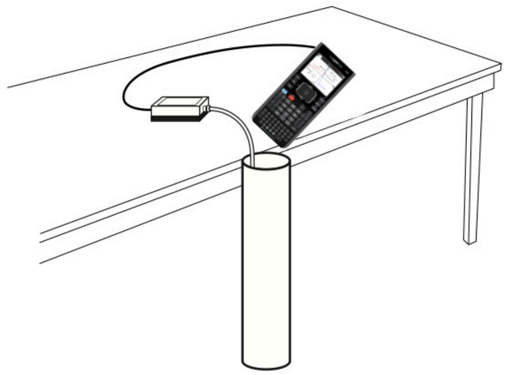
1. Recall that pressure is force per unit area. The units we will use are Newtons (N) for force and Pascals (Pa) for pressure. A Pascal is simply a Newton per square meter. If we assume that for a small change in depth, the pressure is relatively constant, we can approximate the force at that depth by multiplying the area of a strip by the pressure (see figure below). Using this approach, find the area of a rectangular strip located at position y along the y-axis. State your expression for area of the strip of width Δy at the given location, y, along the y-axis.



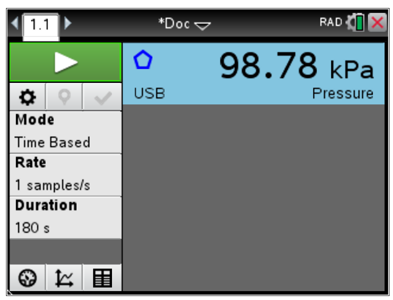
Now that we have a model for the area, we need to model the pressure at varying depths. Using the Vernier gas pressure sensor, you will now collect data and perform a curve fit on the data to get a model for the pressure at a given depth, *d*.

- Make a small hole in one end of the plastic aquarium tubing. Attach a paperclip to the tubing through the hole. This clip will be used as a hanger for the sinker weight.
- Attach the sinker weight to the paper clip. Attach the other end of the plastic tubing to the white stem on the Pressure Sensor.
- Using a permanent marker, mark the aquarium tubing from the end with the sinker attached in 0.25 m increments. This will allow you to lower the tubing to the desired depths when collecting data.

- With the PVC pipe oriented vertically, cap down, secure the pipe safely to a stationary object. Fill the pipe with water, leaving a few cm at the top unfilled to allow for the displacement of the tubing volume as it is lowered into the water.

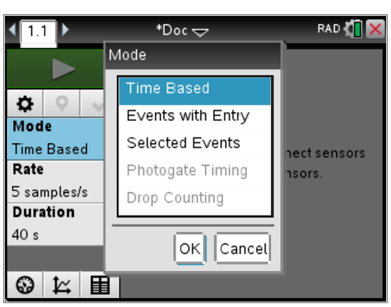


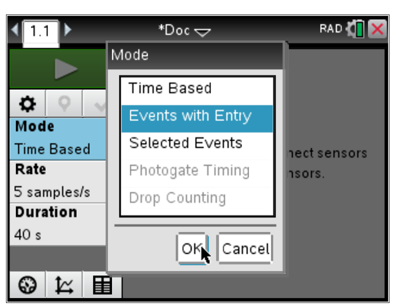
- Turn on the calculator and open a new document. Connect the Pressure Sensor to the calculator. A new Data Quest application will automatically open.

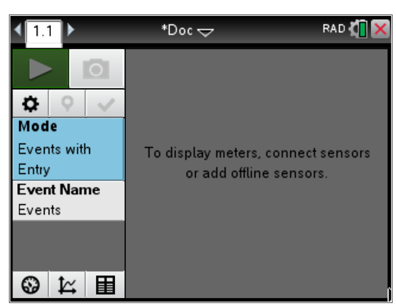


We now need to set up the experiment.

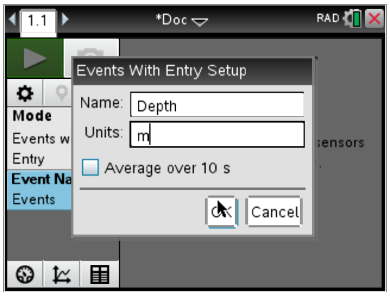
- The default mode is “Time Based” where the device will collect data in a set frequency based on time. For this experiment, we want to control when we take a pressure reading since we will need to lower the tubing to the given depths before sampling the pressure. To do this, we will change the mode to “Events with Entry” so that when we take a pressure reading, we can enter the depth at which the pressure was taken. Click on the Mode setting on the screen. This will open a dialogue box where you can click on **Events with Entry** and then click OK or press [enter] (see images below).

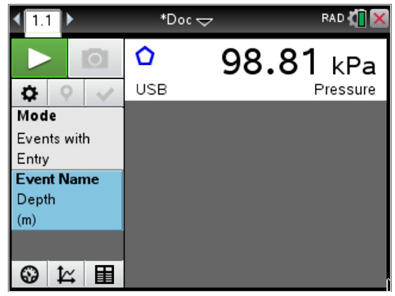




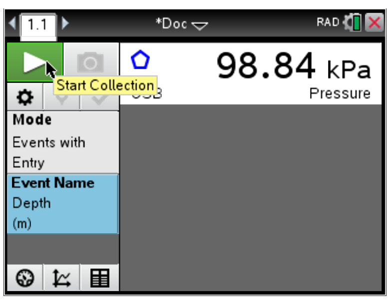


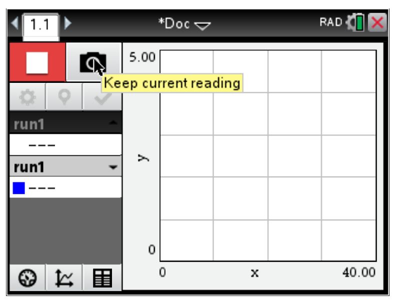
- We can now label the “Event Name” since we will be entering the depth here. Click on the Event Name field and enter the name as Depth and the units as meters and click OK (see below).

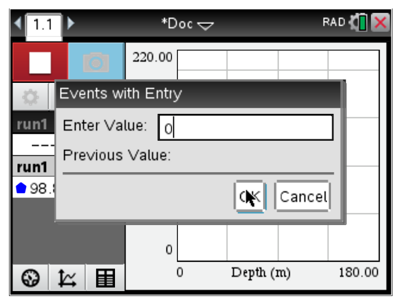




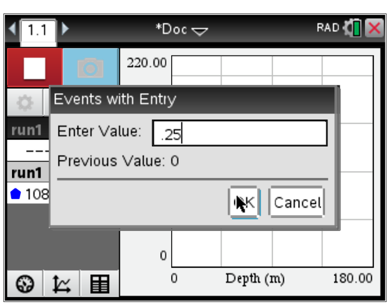
- To begin data collection, click on the “Play” button. To take a pressure reading, simply click on the camera icon. You will want to take your first reading before you lower the tubing into the water since this will be a zero depth. Enter the depth value as 0 and click OK.

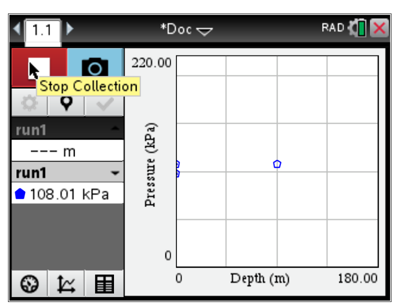




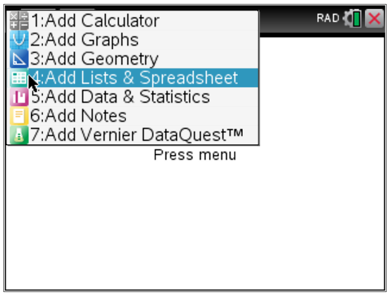


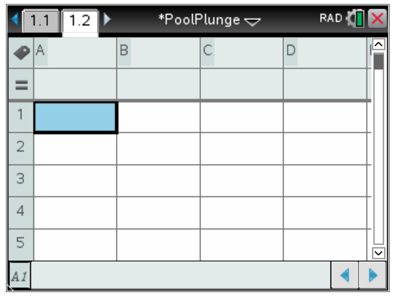
- Now repeat this process by lowering the tubing to depths of 0.25, 0.50, 0.75, 1.0, and 1.25 m and entering the depth value at each stage. When you have finished the data collection, click the “Stop” icon.

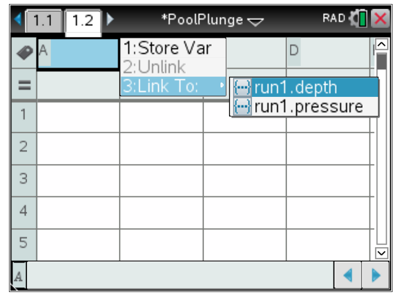




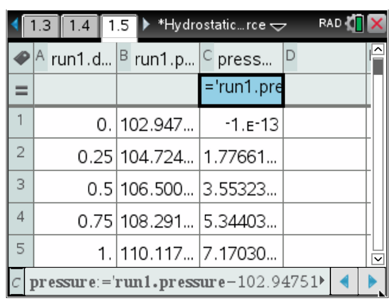
- To explore this relationship, we will place the data into a spreadsheet so that we can work with them more easily. To do this, move your cursor to the very top of the first column and press the var key. Now select **Link To:** and choose *run1.depth*. This will place the data collected during the experiment into the first column. Now repeat this process selecting *run1.pressure* and placing it in the second column.







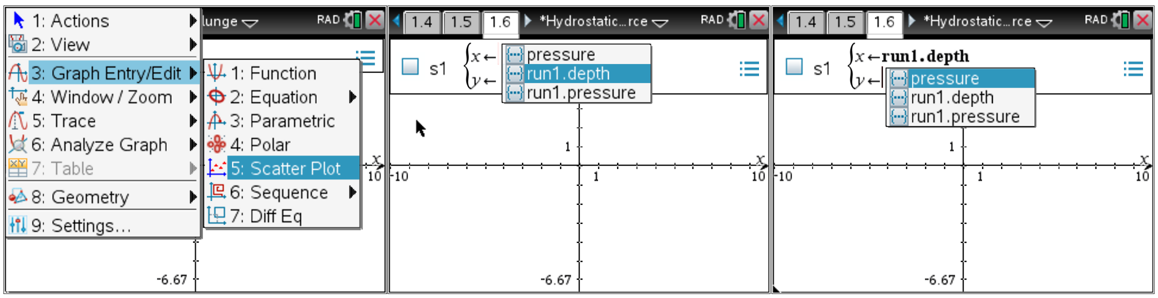
2. Now the pressure we have found is the actual pressure with atmospheric pressure included. If we want to take into account the fact that there will be air pressure on the opposite side of the dam and thus only want the net force *difference*, we need to have the net pressure *difference*. To account for this, create a third column called pressure and subtract the atmospheric pressure (pressure you measured at depth 0) from your pressure readings.



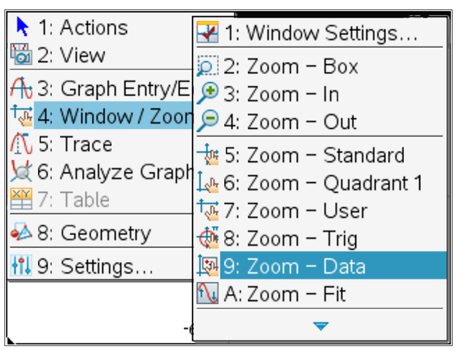
3. Now that you have entered the data into the spreadsheet, record your data in the table below.

x: Depth (m)	y: Pressure (kPa)
0	
0.25	
0.50	
0.75	
1.0	
1.25	

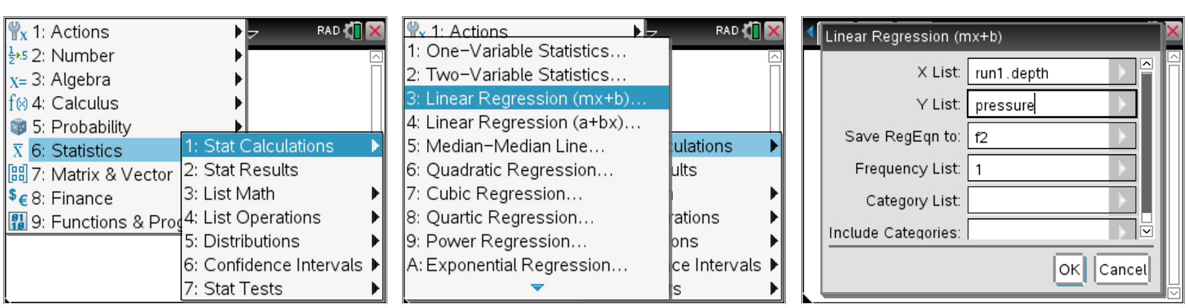
- To find a relationship between depth and pressure, we can now try to find a function that will fit the data. To begin we need to graph the data on a **Graphs** page. Open a new page by pressing ctrl +page and selecting **Graphs**. Now graph the data by pressing menu and selecting **Graph Entry/Edit** followed by **Scatter Plot**. To graph the data, move your cursor to the **x←** line and pressing var select *run1.depth*. Repeat this process placing *run1.pressure* in the **y←** line.



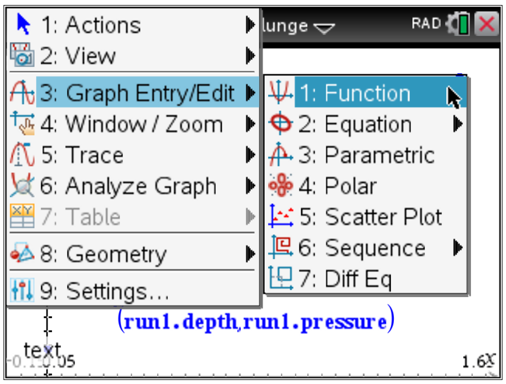
- To see the data in the viewing window, press menu and select **Window/Zoom** followed by **Zoom-Data**. Give a rough sketch of your graph labeling the axes. Describe the relationship you see between the depth and pressure.



4. You should notice a fairly linear relationship between pressure and depth. To find the line of best fit, press ctrl +page and add a **Calculator** page. Now press menu and select **Statistics→Stat Calculations→Linear Regression**. Clicking on the **X List** and **Y List** fields select *run1.depth* and *pressure* respectively and click OK.



- The linear model was stored in the function f2(x). To see it graphed on top of the data, we need to switch back to the function plot by moving to the **Graphs** page and pressing [menu] and selecting **Graph Entry/Edit** followed by **Function**. Move your cursor to the line containing the function f2(x) and press [enter] to graph it on top of the data. Sketch the graph of your data and linear function below.



Give your function: f2(x)=_______________ x+_________________

a. Explain how the values for the coefficient of x and the y-intercept are related to the physical situation.

b. Based on your model, what would you expect the pressure to be at a depth of 10 m, 20 m, and 30 m?

c. Recall that our pressure probe collected pressure data in kPa. For our force calculation, we need to have our units in Pascals (Pa). Give a pressure function, $P(d)$, relating pressure as a function of depth, d, with units of Pa instead of kPa. State it here and define this function in your CAS.

Summing Things Up

5. Now that you have expressions for both area and pressure, we need to set up an approximating sum for the total force. The product of area and pressure will give an approximation for the force on the strip at the location, *y*, along the y-axis. We now need to add all of these forces.

   a. Consider the force on a strip located at the position y_i along the *y*-axis. Give an expression of the force on this particular strip. Keep in mind that you have already defined *P*(*d*) in your CAS and can use it. Also, notice that your function uses depth, *d*, rather than vertical position, *y*. Think about how are *d* and *y* related.

   b. Now we need to sum these forces. Give an expression for the total force (sum of all forces, F_i) for all y locations.

   c. If we assume $\Delta y \to 0$, the sum you found in part (b) becomes an integral. Give the integral for the total force and compute it.

