The Coffee-Turkey Juxtaposition

Materials

- *TI-Nspire™ CX CAS handheld*
- Vernier EasyTemp Sensor
- Hot water source
- Coffee cups
- Freshly cooked turkey

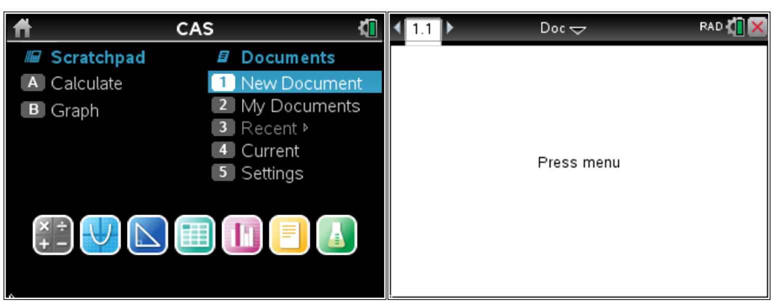
Introduction to the Investigation

Have you ever noticed that when a hot liquid is placed in different environments, the rate of cooling of the liquid is affected? Similarly, if you turn off your grill in the winter, you can almost immediately place the cover on the grill without worrying about the grill temperature damaging the cover; however, in the summer, the same grill requires more time before placing the cover over the grill.

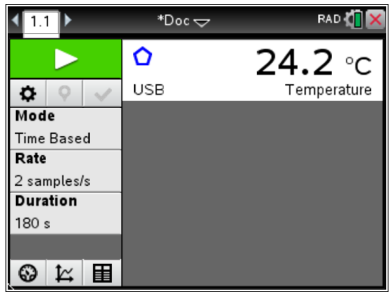
Finding a relationship between temperature and time can be helpful in predicting the way in which objects cool. Suppose you own a company that manufactures paper coffee cups. When coffee or tea is poured into the cup, the customer does not want to wait too long for the beverage to cool to a desired drinking temperature (usually around 75˚C); however, s/he also does not want the beverage to cool too quickly so that it is no longer “hot”. How might we decide what material to use in creating cups so that the hot beverage reaches drinking temperature fairly soon while still remaining warm for a reasonable time? In this activity you will use a temperature probe to explore the way in which the temperature of cooling objects changes over time.

Explore

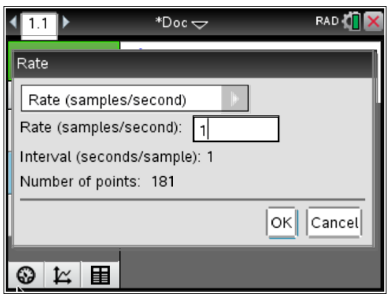
1. Open a new document on your TI-Nspire CX CAS calculator.

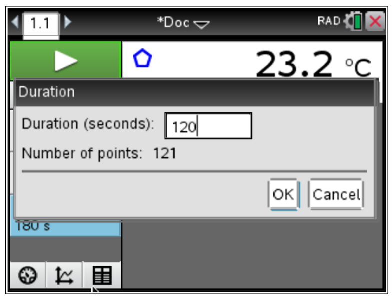


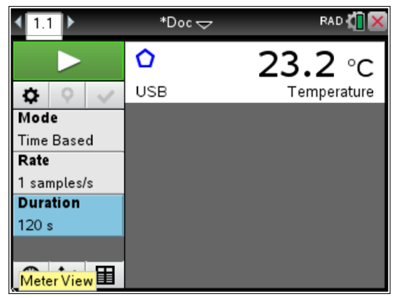
2. Connect the *EasyTemp™* probe to your calculator. The connection will automatically trigger the opening of a *Vernier DataQuest™* application. You will see the temperature of the room displayed on the screen. Record your room temperature: ______________________



3. We will want to collect temperature data for the cooling liquid for 2 minutes (120 seconds) sampling one data point each second. To do this, click on the **Rate** field and enter 1 followed by the **Duration** field and enter 120.

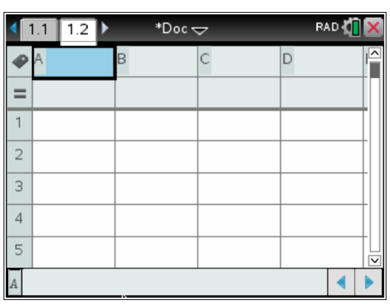


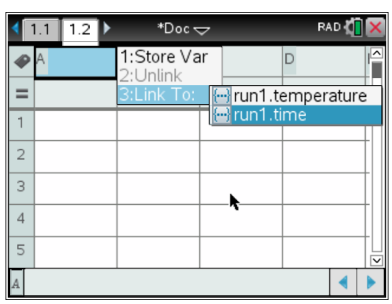


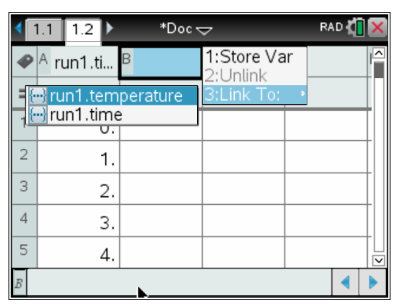


4. We are now ready to collect temperature data of a cooling liquid. Boil water and pour it into the paper coffee cup and immediately place the temperature probe into the water. Observe the temperature value displayed until it reaches a maximum value and begins to cool. When the temperature begins to drop, click the "play" button.

5. Once the data collection is completed, we are ready to look at the data. To give the most flexibility in analyzing the data, we will look at the data in both a spreadsheet and scatter plot. Press ctrl +page and insert a new **Lists & Spreadsheet** page. Move your cursor to the very top of the first column and press the var key. Now select **Link To:** and choose *run1.time*. This will place the data collected during the experiment into the first column. Now repeat this process selecting *run1.temperature* and placing it in the second column.





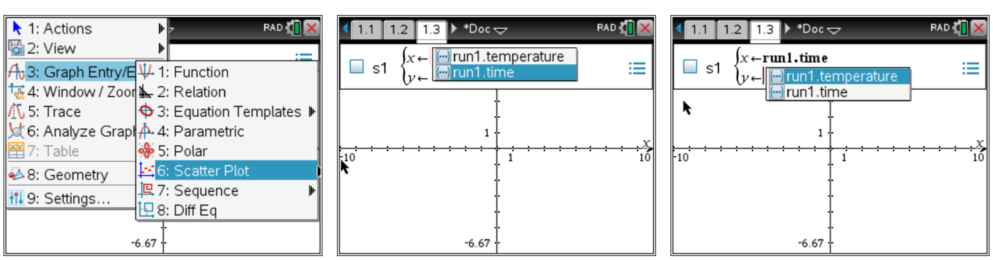


6. Although we collected temperature readings every second, we will just list data below for every 10 seconds so that we can look for patterns. In the third column, calculate the change in temperature between 0 and 10 seconds, 10 and 20 seconds, …etc.

Time (sec)	Temperature (˚C)	∆Temp (˚C)
0		
10		
20		
30		
40		
50		
60		
70		
80		
90		
100		
110		
120		

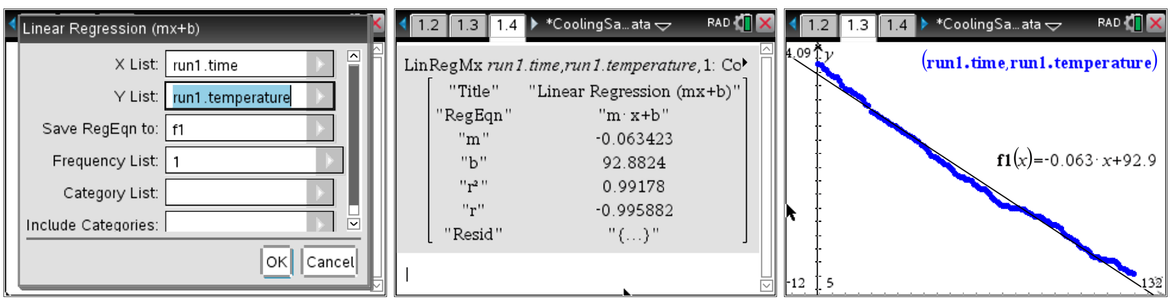
7. Describe any patterns you see in the *change in temperature* as the time changes in 10 seconds intervals.

8. To look at the graphical pattern of the data, press ctrl +page and insert a new **Graphs** page. Press menu followed by *Graph Entry/Edit* and *Scatter Plot*. Press the var key and select *run1.time* for the *x*-coordinate and then *run1.temperature* for the *y*-coordinate.



9. Press [menu] followed by *Window/Zoom* and *ZoomData.* This will fit the data into the viewing window. Sketch your graph below. Describe the data pattern you see.

10. One model we might suspect would fit the data could be linear. To explore this, we can insert a line of best fit on the graph. To do this, open a new Calculator page within the same document and press [menu]. Select **Statistics**→**Stat Calculations**→**Linear Regression (mx+b)**. You will be prompted to enter the X and Y Lists. Using the arrow keys, select *run1.time* for the *X* List and then *run1.temperature* for the *Y* List in the dropdown menus. Pressing [enter] will perform the regression and store the function in the chosen function name.



11. To see the linear function on top of the data, on the **Graphs** page, select [menu] and choose *Graph Entry/Edit* to switch back to the function plot. Move up to f1(x), assuming this is where you stored your function, and press [enter] to display the graph of the function (see above). Record your equation:

____

   a. What does the coefficient of *x* in your equation represent?

b. What does the *y*-intercept of your equation represent?

c. Based on your **equation**, describe how the temperature changes as the time goes up in 1 second increments.

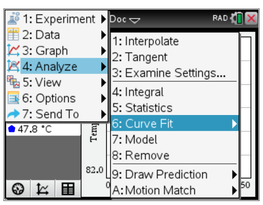
d. Based on your **equation**, describe how the temperature changes as the time goes up in 10 second increments.

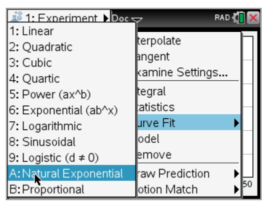
e. Does this pattern match the pattern you found from your data table above in question 6? Explain.

f. What would your equation predict for the temperature after 50 minutes (3000 seconds)? Does this make sense? Is a linear model a reasonable way to describe the temperature-time relationship? Explain.

12. Based on the data from your table in question 6, when did the temperature drop the quickest? When did it drop the slowest? How do these observations fit with a linear model for describing the temperature-time relationship?

13. Since a linear model does not seem to describe the pattern for a cooling liquid, consider another model where the rate of change (*magnitude* of temperature drop) continually gets smaller. This seems to describe exponential decay. Using the same process from questions 10 and 11, perform an exponential regression on your data and plot your result stating your function. Note that you can also do a curve fit in the DataQuest app to get a natural exponential so that your function is of the form Ae^{-kx} (see screens below). Does your model seem to describe your data well? Explain your reasoning.

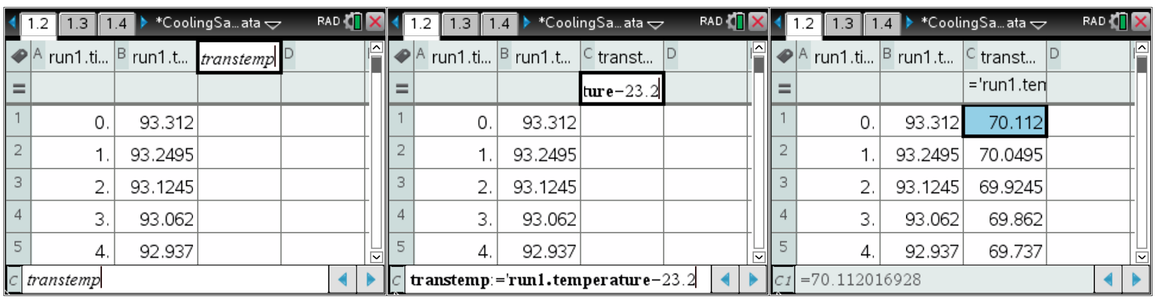




14. Consider your exponential model from question 13. What would your equation predict for the temperature after 50 minutes (3000 seconds)? Does this make sense? Is an exponential model a reasonable way to describe the temperature-time relationship? Explain.

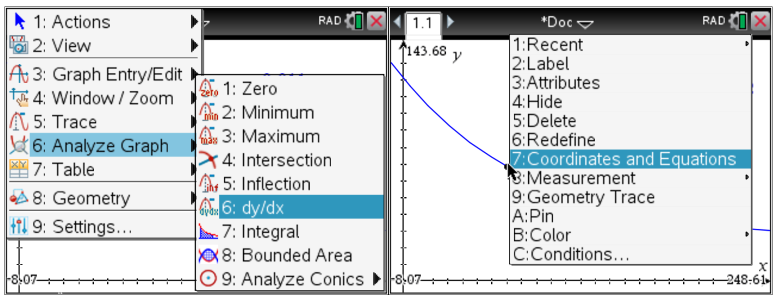
Since an exponential function will approach the *x*-axis in one direction or the other (depending on if it is growing or decaying), we need it to approach room temperature instead. To accomplish this in our curve-fitting process, we must transform the data so that they approach zero and then perform the regression. This essentially shifts the data down by the value of room temperature so that the regression operates on data approaching zero. After we get the model, we simply shift it back up by room temperature so that it falls along the original data.

15. To perform this regression, we need to create a new column in our spreadsheet for the transformed *y*-coordinates of the data. In your spreadsheet, move to the very top of an open column and enter ***transtemp*** for transformed temperature. In the cell directly below the name, we will subtract room temperature from all *y*-coordinates by pressing the var key and selecting *run1.temperature* and then subtracting the room temperature value you recorded earlier (in the example below, I am using 23.2). When you press enter you should see the new *y*-coordinates of your data appear in the column.



16. Using the same process as before, perform an exponential regression on your data using *run1.time* and *transtemp*. If you try to plot this model, you will notice that its graph will be below all of your data (precisely by the value of room temperature). To make your model fit the original data, you will need to add room temperature to the end of the function to shift it back up. Do this and plot your result stating your new temperature function. Does your model seem to describe the data well? What would your new shifted exponential model predict for the temperature after 50 minutes (3000 seconds)? Does this make sense? Is a shifted exponential model a reasonable way to describe the temperature-time relationship? Explain your reasoning.

17. Now we would like to answer the question, what does the relationship we found between the time and temperature say about the relationship between the rate of change of the temperature and the temperature itself? Using your model for temperature vs. time, trace along the graph and record the temperature and rate of temperature change ($\frac{dy}{dx}$) at various points on the graph. To do this, from the Graphs page, press [menu] and then *Analyze Graph* followed by *dy/dx* and click on a point on the graph. You will see the numerical value of slope given on the screen. To see the coordinates of the point, simply press [ctrl] [menu] and select *Coordinates and Equations*. This will allow you to see the y-coordinate (temperature value) and the rate of temperature change (slope) so that you can record the values.



Temperature	Rate of Temp Change

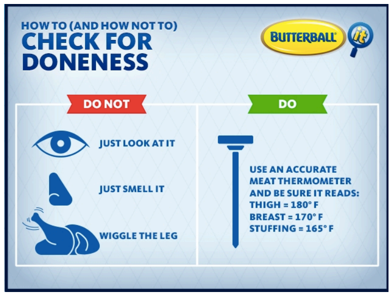
18. Now plot your data from the table above with temperature on the horizontal axis and the rate of temperature change on the vertical axis. Describe the type of relationship you see and apply a curve fit that is appropriate for this relationship sketching the model and data below. Explain any patterns you see between the original exponential model (of the form $Ae^{-kx}+T$ with T as room temperature) for *temperature vs. time* and the model you obtained for the *rate of temperature change vs. temperature*. Justify why your observations make sense using what you know about the derivative of exponential functions.

19. On the window [-50, 1200] by [-3, 110], now plot the slope field for the equation you obtained in question 18 using y1' as your "*dy/dx"* axis variable and y1 as your "*y*" variable. To do this, press menu and select **Graph Entry/Edit** followed by **Diff EQ**. Describe how the shape of the slope field relates to the behavior of the graph of the function you found in question 18?

Coffee-Turkey Juxtaposition

Analyze & Apply

We will now take our observations and apply them to a decision for eating turkey. In order to be safe for consumption, according to Butterball, when cooking a turkey, the breast should reach 170˚F (or 76.66˚C) before pulling it from the oven.



20. In front of us, we have a cooked turkey that was removed from the oven/roaster a period of time ago, but unfortunately, we did not check the temperature at the time it was removed from the heat source. Your job is to decide if it is safe to eat. To do this, we will collect temperature data for 10 minutes and you will have to base your decision on these data along with the knowledge of how long ago the turkey was pulled from the heat source. Below, at the time we begin the temperature data collection, record the time that the turkey has been out of the oven/roaster along with the room temperature. The temperature data will be shared with your group once data collection is completed.

    Time since removed from roaster: ______________________

    Room Temperature: ______________________

21. Using all of the data and information known to you, is the turkey safe to eat? Explain your reasoning to convince someone whether or not you would eat the turkey. Provide the graph of your data below. You may want to consider the behavior of your cooling liquid and when the temperature was changing the most/least.