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Early Secondary Mathematics

Unveiling Student Understanding: The Role of Questioning in Instruction

WE BEGIN THIS ARTICLE BY PRESENTING A vignette from a ninth-grade algebra 1 class:

Laura, the teacher, had finished teaching a section on solving systems of linear equations using the method of elimination. After reminding her students of the method of substitution that they had learned the previous day, she used two examples to illustrate how each of the methods could be employed to find the values of the two unknowns. Before assigning exercises for in-class practice, she asked whether students had any questions about either of the methods that she had discussed.

Teacher. Do we all understand the difference between the two methods?
[*Silence, some students nod their heads.*]
Teacher. Does anyone have any questions?
Sam. How many problems on the worksheet?
Teacher. About ten—but first I want to make sure that we are clear on the two methods. I do not want you to hold back if you are not sure about what we did here.
[*Silence.*]
Teacher. Jacob? Are you clear on what we did? Any questions?
Jacob. No.
Teacher. Susie? Everything okay?
Susie. Um . . . can we look at another example?
Teacher. [*Looks back at the chalkboard*] I am glad you asked, Susie. Which do you want to see again: elimination or substitution?
Susie. It doesn't matter.
Teacher. I think that the hardest one for most people is the elimination method—so, let's do another one of them. Who wants to give me the first equation? Joshua?
Joshua. $2x + 3y = 10$.
Teacher. [*Writes the equation on the board*] Okay, next one? Travis?

Give us your favorite equation with two unknowns

Travis. Um . . . do you want it to have a fraction?
Teacher. I don't care—just give us your favorite linear equation with two unknowns! [*Everyone laughs.*]
Travis. $10x + 4y = 25$.
Teacher. Good, Travis. Now, let's see. Here we have $2x$ [*points at the first equation*], and here we have $10x$ [*points at the second equation on the board*]. What we need to remember is that we want to get rid of one of the variables. Which one do you want to eliminate first?
Travis. x?
Teacher. How about the rest of you? Are you okay with eliminating x first?
[*Everyone nods in agreement.*]
Teacher. Okay, now we have to make the coefficients of x be equal in both equations; notice that one is 2 and that the other one is 10. By what number should we multiply the first equation to make the coefficient equal 10?
Chorus. 5.
Teacher. Excellent—now when we multiply the first equation by 5, what do we get? Remember that we have to multiply every one of the terms in that equation by 5, not just the first one. What do we get?
Susie. $10x + 15y = 50$. [*The teacher writes Susie's equation on the board.*]
Teacher. Good! The next step is to subtract one from the other. Do we know why we subtract?
Patrick. We want to get rid of the x?

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Teacher. Very good! Since the coefficients are equal once we subtract them, it eliminates the x's. [*She subtracts the second equation from the first one and writes* $9y = 50 - 25$.] What is $50 - 25$?

Arthoro: 25.

Teacher. Is he right? Do we agree that it is 25?

Chorus. Yes.

Teacher. Good! Now what we get basically is a linear equation in one variable. We know how to solve these babies, right? [*She writes* $9y = 25$.] Tommy, you have been quiet all day.

Tommy. Divide by 9.

Teacher. Just the right-hand side of the equation?

Tommy. No, both sides . . . um . . . I mean divide both sides by 9.

Teacher. Why do we divide by 9?

Tommy. Cause we have 9. If it were 2, we would divide by 2.

Teacher. Good—and what do we get? What is 25/9?

[*Silence.*]

Teacher. We can even leave it like that. I don't want you to get bogged down with the fractions right now. Just think about the process. The process is the important thing here for now. Now that we have the value of y, what do we do next?

Susie. We find x.

Teacher. Right, Susie. Now that we have y, we can substitute it in either one of the equations and find x. Do we all see that?

Joshua. But I thought you said this was the elimination method; how come we are substituting?

What are your impressions of the vignette? How would you categorize the teacher's questions? How do you categorize the students' responses? What can you say about what students knew or learned by the end of the episode?

We can make a number of observations about the teacher's actions, as described in the vignette. For instance, we can clearly see that Laura, the teacher, tried to assure that her students knew the steps for solving linear equations in one or two variables. She reviewed procedures when students requested additional examples. She insisted that students ask questions. She purposefully called on specific students to ask or to answer questions. She also tried to assess whether students remembered the algorithms and methods that she had discussed in previous lessons.

We can, however, draw limited conclusions about what the students could or could not do at the end of the session on the basis of their "talk," or their responses to the questions that Laura asked. The discussion does not show enough evidence of students' work and thinking or what they learned or did not learn about the central topic of the lesson. Although Laura posed numerous questions, her questions seemed to control the students' answers. Thus, the students' responses do not reveal much information about the nature of their understanding, misunderstandings, and competence in either the computational or conceptual domain. Almost all Laura's questions called for remembering isolated skills and procedures. These questions did not inquire whether her students understood why or when certain procedures were used. Most of Laura's questions, even those that appeared to be process-oriented, solicited dichotomous responses from the students. Student "talk" remained vague and did not disclose much information about their thinking. Even when one of the students (Susie) asked whether Laura could demonstrate another example, we remain unclear about the difficulty that Susie was experiencing or what she hoped to learn from the new example. Laura did not try to detect the problem area. In fact, Susie's response ("It does not matter"), which should have been perceived as a warning signal and should have prompted Laura to ask her to say more, remained unexplored. Laura's next teaching move was based on her assumptions about what could have been the source of difficulty rather than what she learned about Susie's need.

Joshua's remark ("How come we are substituting?") at the end of the session was revealing because it highlighted his confusion on the most fundamental point of the lesson. Perhaps Joshua perceived that after the method of elimination was used to find the value of one of the unknowns, substitution was not a suitable approach for finding the second unknown. How many other students had a problem similar to Joshua's? How many of them would have used a different procedure than the one that the teacher suggested for finding the value of the second unknown? How many of them fully understood the processes that the teacher emphasized in the session. Neither the type of questions that the teacher asked nor the students' responses allow us to assess the substance of the students' learning. Effective instruction includes question types that can provide the teacher with such information.

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THE ROLE OF QUESTIONING IN INSTRUCTION

One of the most striking aspects of teaching is that the teacher's speech consists of questions. These questions are central to the type of learning that takes place in the classroom. Naturally, questions are built around varying forms of thinking. Some questions are aimed at recall of information, whereas others provoke problem solving or concept development. In a general sense, teachers' questions control students' learning because they focus students' attention on specific features of the concepts that they explore in class. Moreover, these questions establish and validate students' perceptions about what is important to know to succeed in mathematics class.

Traditionally, questions have been used to determine what has been learned—too often as isolated bits of knowledge. Building procedural skills is no longer the sole purpose of mathematics instruction. The current emphasis on processes of mathematics expects students to practice reasoning from the data, learn to argue a point of view, and examine mathematics from more than one perspective (NCTM 2000). Teachers should include questions that are directed toward evaluating students' thinking. The teacher's questions must give learners an opportunity to communicate their reasoning processes. These types of questions allow the teacher to gather detailed data on how students think and what they actually learn from instruction. In fact, several teachers and educators have reported instances from mathematics classrooms when students successfully gave correct answers to problems; however, the depth of their misunderstandings or the nature of their misconceptions became obvious only when they were asked to explain their thinking (Wagner and Parker 1993). These educators suggest that unless students are asked to explain their thinking, a teacher may not know which concepts the students understand.

The teacher's questions must give learners an opportunity to communicate their reasoning processes

An integral aspect of effective instructional planning is determining the questions to pose in class. Asking good questions is a sophisticated skill that needs practice and thoughtful planning, as well as reflection on and analysis of the mathematical and pedagogical goals of lessons.

ANALYZING QUESTIONS: FORM, CONTENT, AND PURPOSE

In designing questions, the teacher should consider several important issues. They relate to the form, content, and purpose of questions.

Form

The form of the question determines the type of answer that the teacher obtains from the students. A question can be posed in a *closed form* to seek a particular answer. These questions usually are stated to solicit dichotomous right or wrong or true or false answers. In contrast, questions posed in *open form* (for example, that begin with *how* or *why*) are aimed at promoting a description of a certain type of solution method or strategy or a process that enables the students to find some answer. We next consider examples of closed-form and open-form questions that address the concepts presented in Laura's classroom.

1. "Does everyone understand the method of elimination?" as opposed to "When is using the elimination method in solving systems of linear equations more advantageous than using other methods? Why does elimination yield a solution?"
2. "Are you clear on the difference between the methods of elimination and substitution?" as opposed to "What should you consider when deciding which method to use in solving instances of systems of linear equations? How do you decide which method is more efficient?"
3. "What is the next step?" as opposed to "What could you do next and why? How could you proceed from here? How do you know that the solution you find from elimination is a solution to both equations?"
4. "Is this statement true or false?" as opposed to "When is this statement true, and when is it false? How do you know?"
5. "Does anyone have any questions about what we did?" as opposed to "What are some good questions to ask about what we discussed today?"
6. "Is this clear to everyone?" as opposed to "Identify three features of this process that are most clear to you."

Although each question form is useful, depending on the teacher's specific objective, the preceding examples show that the two categories of questions elicit different types of information from students. Indeed, each question form forces students to engage in a different kind of thinking about, and relationship with, the mathematics content being studied. In the following section, we elaborate on that point.

Content

The content of a question is most critical in the teaching process. The question content determines the type of information that a teacher obtains about students' thinking. The content embedded in questions may range from applying a specific mathematical concept (for example, solving a well-defined word problem that encourages practice of a specific skill) to engaging students in a purely exploratory investigation. Both implicitly and explicitly, the content of a question is driven by a teacher's sense of what is important for learners to know and be able to do.

We next reexamine the content of questions posed in the previous section. The first question in example 1 is stated in a closed form. At the outset, it has the potential to determine the number of people who claim understanding of a piece of mathematics. The same is true for the first question in example 2, stated in closed form. The contrasting questions in each example, however, ask students to analyze and evaluate various methods. To answer these questions, the students need to pay attention to both the content and the context of the methods discussed in class and to assess specific features of each method. Similarly, examples 5 and 6 posed in open-ended form can give the teacher a greater knowledge base about his or her students' thinking, their conceptual development, and their level of comfort with the targeted concepts and algorithms. The teacher can then organize instruction to meet the specific needs of the group. In the closed-form question presented in example 3, students are expected to remember specific procedures, whereas the contrasting question allows them to identify other methods that appear meaningful to them. In this example, the teacher has a greater chance of determining whether the students understand why specific procedures are emphasized in the process. Determining when and where to use each type of question depends on the purpose of each question and on the mathematical and instructional goals of the teacher.

Purpose

Some questions are posed to engage inactive students in the activity. Some questions are aimed at testing students' mastery of specific skills. Others try to encourage students to explore mathematical relationships and to build connections among topics discussed in class. Some questions are posed to create a sense of community and to build group relationships among students. Others are posed to establish individual accountability and to detect individual progress.

The multiple ways in which questions can assist a teacher in advancing pedagogy place greater emphasis on the need for planning the types of questions that should be used in instruction. Both long- and short-term instructional goals must influence the questions that are posed in class and their frequency. That is, if the teacher's goal is to develop problem-solving skills among his or her students or to help them develop an appreciation for both the beauty and the usefulness of mathematics, then he or she must regularly ask the types of questions that foster an exploratory disposition toward the study of mathematical concepts. Asking questions that measure students' mastery of basic skills is certainly important, since answers to such questions give useful information on students' procedural knowledge; however, these questions are not adequate in determining what students can do beyond solving exercises. A problem such as "Solve: $x^2 - 2x + 2 = 0$" coupled with "Can we find another quadratic equation whose roots are the same as those of $x^2 - 2x + 2 = 0$? How do you know?" provides a powerful means for a teacher to obtain data on the students' skills and the depth of their understandings. These data allow the teacher to develop a coherent profile of what learners know and whether they fully grasp the concepts explored in class. When teachers are pressured by the need to place immediate closure on class discussions, they often rely solely on closed-form questions that assess learners' mastery of isolated skills and knowledge. Effective instruction strives to take advantage of information obtained from all question types to improve learning. Building questions that assess both skills and conceptual understanding in one question gives the teacher a better understanding of students' knowledge.

SUGGESTIONS FOR INSTRUCTION

One of the most important strategies for effective questioning is to identify, in advance, the big ideas that the lesson examines and the mathematical outcomes that students can achieve. In planning instruction, the teacher must consider several questions. They include the following:

- What do I want the students to know at the end of this lesson or unit, and how do I know whether they really know it?
- How does this new concept relate to the ones that the class has discussed, and how do I assess whether the students realize the connections?
- What are some misconceptions about the concept that I am teaching, and how can I determine whether my students have these misconceptions?
- If my goal is to measure the different layers of student understanding of this concept, what questions should I ask them?
- What should I ask to help students focus on similarities and differences among various methods and techniques?
- What questions can I ask that will allow me to determine whether students can use the procedure in context? How do I determine whether they can use the procedure in a novel situation without my telling them?
- How should I phrase the question to meet the needs of students of various abilities?

Identify, in advance, the big ideas to be examined

Thinking through these questions allows the teacher to prepare thoughtful lessons that help students discover and engage in mathematical inquiry. It also makes teaching exciting, since it provides opportunities for the teacher to experiment with ideas, learn more about his or her students, and discover their true potential and capabilities.

ADDITIONAL RESOURCES

Many resources can assist teachers in planning open-ended and thoughtful questions. *Assessment in the Mathematics Classroom* (Webb and Coxford 1993) includes examples that indicate how various educators and teachers use alternative types of questions and tasks in class. In *The Art of Problem*

Posing, Brown and Walter (1983) highlight the centrality of question asking in the development of mathematics as a discipline. Their work gives a useful framework for learning how to generate question types that provoke high levels of mathematical thinking. In addition, Standards-based textbooks offer significant learning opportunities for both learners and teachers by organizing lessons and activities that build around open-ended questions. Information about these textbooks is available at Show-Me Center (showmecenter.Missouri.edu). These resources provide a coherent structure for planning instructional tasks that are both intellectually rich and enjoyable.

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