

MTH 256 Study Guide 1

Solving algebra problems with Algebra Pieces. Make sure that your solution shows how you to use the pieces to find the answer (as opposed to solving the problem using an equation). Review the problem solved in class and the following:

1. A fence around a rectangular lot is 24m long. One side of the rectangle is three times longer than the other. What are the dimensions of the lot?
2. For every 3 girls on a playground, there are 4 boys. There are 28 children on the playground. How many girls and how many boys are there?
3. A piggy bank has \$2 in it. There are twice as many nickels as dimes. How many nickels and how many dimes are there?
4. The sum of five consecutive numbers is 45. What are the numbers?

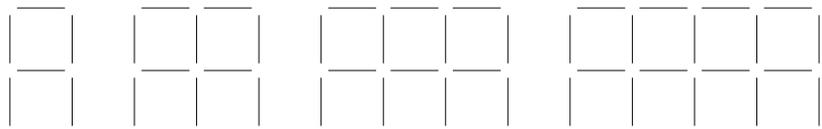
Foundations of algebra: Operation properties

5. Define or describe the properties of operations and give examples to illustrate them. Commutative, Associative, Identity, Inverse, Distributive property. Explain closure property. Discuss a property of multiplication with respect to 0.
6. Simplify the following expressions indicating the properties used:
 - a. $15x + 4x$
 - b. $6 \cdot (3x^2y)$
 - c. (factor) $12x^2yz + 4x^3y^2 - x^2y$
 - d. $7x + 7 - 3x$
7. Decide if the following are true. If they are, explain why (you may use the properties or other ways of justification). If they are false, explain how to fix them (and why).

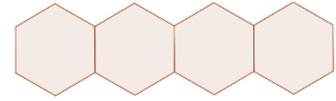
<ol style="list-style-type: none">a. $4(x+7) = 4x + 28$b. $17y + 3y = 20y^2$c. $4(x \cdot 3y) = 4x \cdot 12y$d. $14m - m = 14$e. $S - nS = S(1 - n)$f. $(12x)(2x) = 24x^2$	<ol style="list-style-type: none">g. $\frac{4}{3x} + \frac{3}{3x} = \frac{7}{6x}$h. $\frac{4x+8}{2+x} = \frac{4x}{2} + \frac{8}{x}$i. $\frac{4x+8}{2+x} = \frac{4(x+2)}{2+x} = 4$j. $(x + y)^2 = x^2 + y^2$
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8. Guess my number
 - a. Think of a number. Add 3, multiply by 6, subtract 9 and finally divide by 3. If you tell me the result, I'll quickly tell you your original number. Explain what I do with the result to guess your original number?
 - b. Explain why the following gives the original number as the result: Think of a number. Add 8, multiply by 4, subtract 3, add 7, divide by 4 and subtract 9.
 - c. Invent your own "guess my number" game with at least 4 steps and explain how to quickly find the original number.

Algebra in patterns. Review the problems discussed in class. Here are more to practice:

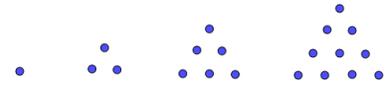
9. How many toothpicks will you need to create the 100th and nth shape in this pattern?



10. Hexagonal tables are arranged as in this picture. How many participants can be seated at 15 tables? N tables? How many tables you will need to seat 86 participants? N participants? (We assume that one participant can seat at each side of the hexagon, except where the tables are joined).



11. The picture explains why 1, 3, 6, etc. are called triangular numbers. What is the n th triangular number?
12. What would be the n -th square number if the first square number is 1?



13. Fill in the following numerical patterns:
- $1, -1, -3, -5, -7, \dots, \dots, \dots, \dots$
 - $3, 9, 27, 81, \dots, \dots, \dots, \dots$
 - $10, 7, 12, 9, 14, 11, 16, \dots, \dots, \dots, \dots$
 - $1, 4, 9, 16, 25, \dots, \dots, \dots, \dots$
 - $1, 2, 6, 13, 23, 36, \dots, \dots, \dots, \dots$
14. Find the sum of the first 10 terms in problems a) and b).
15. Find the digit at the 202nd decimal place of $\frac{1}{13}$.

Arithmetic and geometric sequences. Review the problems solved in class plus the following examples.

16. Derive an explicit formula for
- the n th term of an arithmetic sequence
 - the n th term of a geometric sequence
17. Derive a formula for the sum of the first terms of
- An arithmetic sequence
 - A geometric sequence.
18. Decide if the sequences given by the following explicit formulas are arithmetic, geometric or none of the above. Explain your answer.
- $a_n = \frac{1}{n}$
 - $a_n = 3^n$
 - $a_n = 3n + 2$
19. Find the 500th number in the sequence $7, 13, 19, 25, \dots$. What is the sum of these 500 numbers?
20. Find the 8th number in the sequence $16, 8, 4, 2, \dots$. What is the sum of these 8 numbers?
21. Find the 25th number in the sequence $3\frac{2}{3}, 4\frac{1}{3}, 5, 5\frac{2}{3}, \dots$. What is the sum of these 25 numbers?
22. Find the 7th number in the sequence $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54}, \dots$. What is the sum of these 7 numbers?
23. Find the arithmetic sequence in which $a_1 + a_5 = 30$ and $a_3 + a_4 = 36$.

Algebra as generalized arithmetic

24. Using analogy with adding and multiplying numbers, explain how to add and multiply the following polynomials:
- $P_1: 4n^2 + 3, P_2: 2n + 3$
 - $P_1: 5y^3 + 2y^2 + y + 3, P_2: 3y - 2$
25. Use the Algebra Tiles to explain your answers in part a. (previous problem).
26. Use the Algebra Tiles to perform and explain the following:

Multiply:

- $2x \cdot 2$
- $-2(x+1)$

c. $(2x+1)(x-1)$

Factor:

- d. $x^2 + 3x + 2 =$
- e. $x^2 - 6x + 9 =$
- f. $x^2 + x - 2 =$
- g. $6x^2 + x - 2 =$

Expand:

- h. $(x + y)^2 =$
- i. $(x - y)^2 =$

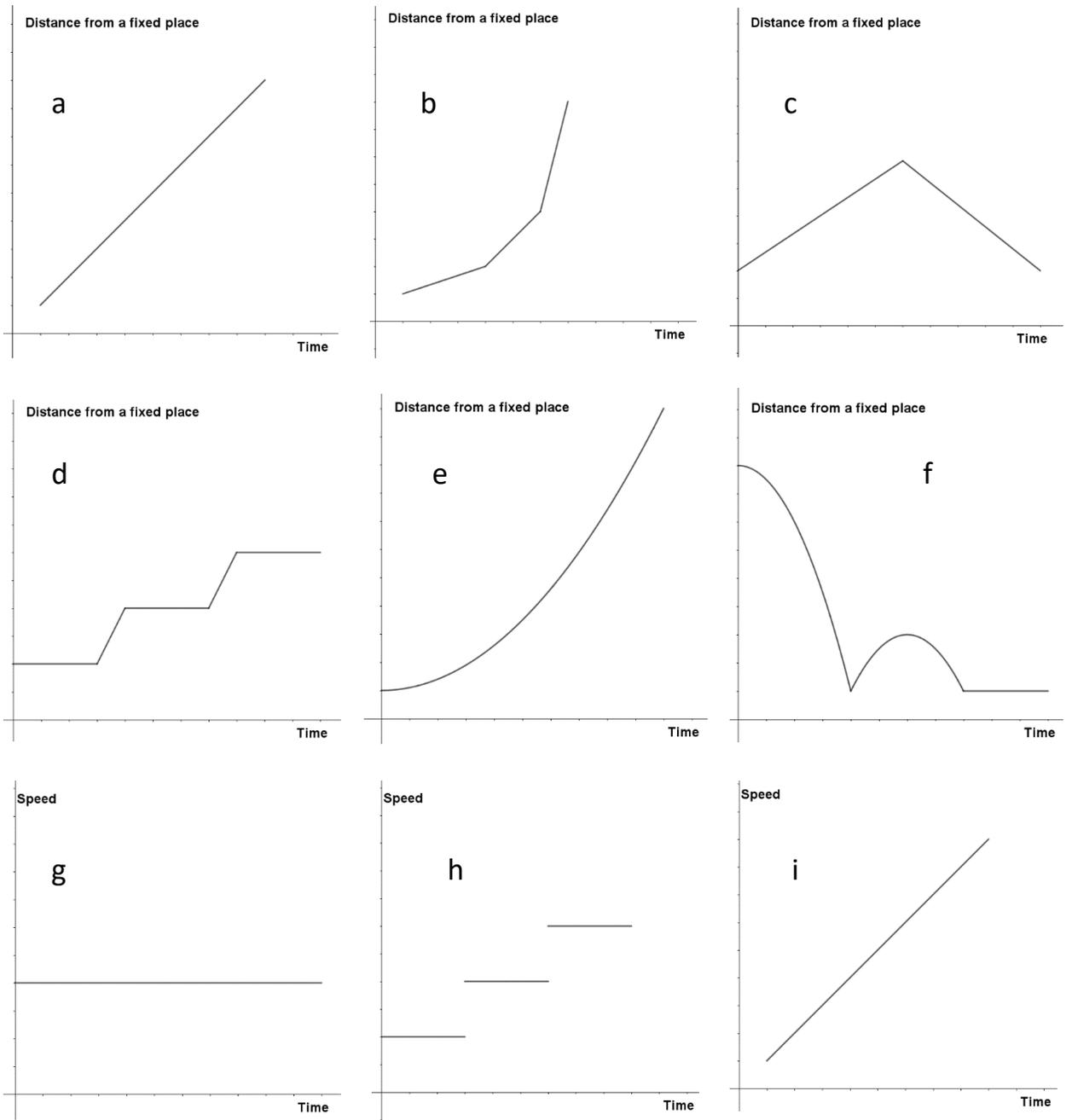
Expand the following two expressions and explain the answer. You may use any method, not necessarily the Algebra Tiles.

- j. $(x + y)^3 =$
- k. $(x - y)^3 =$

- 27. Divide the polynomials $2x^2 + 7x + 6$ and $x + 2$ and explain how this division is similar to a long division of numbers.
- 28. Divide the polynomials $3x^2 + 3x - 6$ and $x + 4$ and explain how this division is different from a long division of numbers.
- 29. Divide:
 - a. $(x^3 - 4x^2 + 2x - 3) : (x + 2) =$
 - b. $(x^3 + 3x^2 - 4x - 12) : (x^2 + x - 6) =$
 - c. $(3x^3 - 2x^2 + 6) : (x^2 - 1) =$
 - d. $(6x^3 + 14x^2 - 2x - 10) : (2x + 2) =$

Functions

- 30. Give an example of a correspondence that is not a function and explain why it is not.
- 31. Sketch the graphs of the following functions:
 - a. $y = x$, $y = -x$, $y = 3$, $y = 3x + 2$
 - b. $y = x^2$, $y = 3 - x^2$, $y = (x - 4)^2 - 2$
 - c. $y = \frac{1}{x}$
 - d. $y = 2^x$, $y = \left(\frac{1}{2}\right)^x$,
 - e. $y = \sin x$
- 32. In your own words, explain what a slope is (not how to calculate it).
- 33. Write down a formula for a linear function that:
 - a. Goes through points (1,4) and (2,1)
 - b. Has the slope of $\frac{1}{2}$ and goes through the point (6,5).
- 34. Can a line be drawn the slope of which is zero? Explain why or why not. If your answer is affirmative, draw such a line.
- 35. Can a line be drawn that “does not have a slope” (e.g. its slope is undefined)? Explain why or why not. If your answer is affirmative, draw such a line.
- 36. A start-up company has 30 employees and their business is growing fast. They plan to hire 5 people a month in the next 10 months, 10 people each month the following 10 months and 15 each month after that. Their target is 300 employees.
 - a. How long will it take the company to grow to their target size?
 - b. Sketch (hand drawing is fine) a graph that describes this information.
 - c. Write the piecewise function capturing this relationship and draw a precise graph on geogebra.
- 37. Describe in words how to walk with a distance sensor to come up with the following graphs. Please pay close attention to the axes' labels.



In the exercises 40 - 42, sketch given graphs into the printed graphs above.

40. Sketch a "Total Distance Traveled" to Time graph for c) and f).
41. A negative value of speed indicates that the motion is in the opposite direction. Sketch a Speed-Time graphs for a-f (f will be challenging!).
42. Sketch a Distance to time graph for g) – i)