General comments. The exam will take place in the Computer Lab. Any construction will have to be saved as a ggb file and sent to my e-mail account. (I will be answering technical questions, make sure you know the math). As to your proof write-up, it does not have to meet “textbook” or “journal” standards but it should be coherent and consistent. To ensure that, always re-read your proof and ask yourself:

- Is each claim (statement) you wrote supported with a clear justification?
- Are there any leaps in reasoning? (Assumptions or statements you are considering without mentioning them).
- Does your conclusion match exactly what you wanted to prove?

More triangle geometry

1. You should be able to construct all 4 triangle centers (Centroid, Orthocenter, Circumcenter, Incenter).
2. You should be able to construct Circumscribed and Inscribed circles to a given triangle (keep in mind that incircle construction has an extra step).
3. Referring to the properties of perpendicular bisector, explain why the circumcenter is constructed as an intersection of perpendicular bisectors.
4. Referring to the properties of angle bisector, explain why the incenter is constructed as an intersection of angle bisectors.
5. Describe or define the Feuerbach circle.
6. You should be comfortable using the trace and locus tools in GeoGebra. Explain how you can draw a Feuerbach circle as a locus of (a) certain point(s).
7. Triangle ABC has the orthocenter O. What is the orthocenter of the triangle ABO? Prove it.
8. Triangle ABC has the orthocenter O. Describe the relationship between the Feuerbach circle of ABC and Feuerbach circle of BCO. Justify your answer.

Similarity

9. When are two figures (not necessarily polygons) similar?
   - Describe the concept on an intuitive level. How do students often describe similar figures?
   - Describe the concept mathematically.
10. When are two polygons similar?
    a. Does the definition of similar figures from the previous question hold for polygons?
    b. Does your definition of similar polygons work for other geometrical figures?

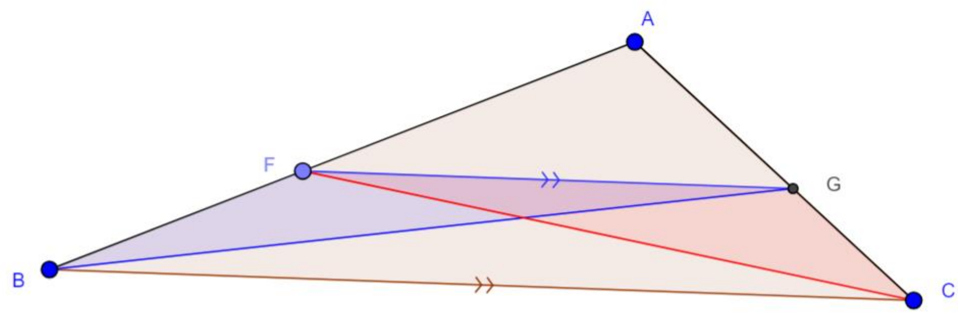
Triangle similarity conditions

11. Define similar triangles.

12. List triangle similarity conditions.
13. How are SAS and SSS congruence theorems same and different from sas and sss similarity theorems? (You don't need to state the theorems, just point to the similarities and differences).
14. Why no asa or aas similarity theorems are mentioned in geometry textbooks? Would they work?
15. Can we use the phrase "CPCTC" for similar triangles? How can the phrase be changed to reflect what's important for similarity considerations?
16. Formulate and prove the Side-splitter theorem. https://www.geogebra.org/m/PdGTJvDC

Note: On the exam, I will not ask you to produce the whole proof but I may ask questions regarding a particular step in the proof. For example a question can be phrased as follows:

In the proof of Side-splitter theorem, the picture below is used to justify a claim about triangles FGB and FGC. State the claim and justify (prove) it.



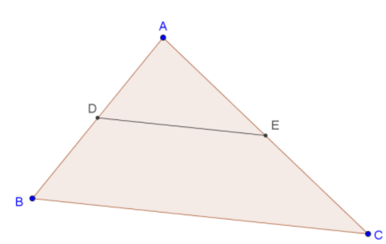
17. Formulate and prove corollary to the Side-splitter theorem.
18. Briefly discuss the importance of the Side-splitter theorem (and/or its corollary).
19. Prove the aa triangle similarity condition.
    https://www.geogebra.org/m/fzpjVwXK#material/WT6nJZ6V

Note: You don't have to repeat the same reasoning in your proof. If some arguments in your proof are repeating, you will be allowed to use "shortcuts" on the exam. Such shortcuts are typically introduced by "Similarly, …". For example, if you are asked to prove that all pairs of angles α,β, and γ are congruent your proof can have this structure:

i. Draw an appropriate picture and use full reasoning to prove that $\alpha = \beta$.
ii. Draw an appropriate picture that applies to β and γ but instead of repeating complete reasoning as in 1., simply state "Similarly (see picture 2), $\beta = \gamma$"
iii. Proceed with other steps or formulate your conclusion.

Triangle similarity applications

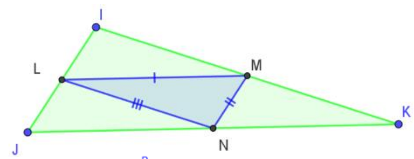
20. D is the midpoint of AB, E is the midpoint of AC. (The segment DE is called *midsegment*.).



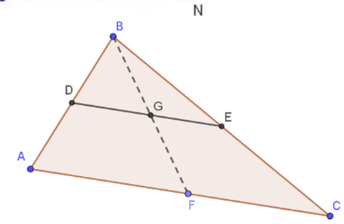
a. Identify similar triangles in the picture and provide proper justification.
b. Formulate two observations about the midsegment (known as the midsegment theorem) and prove them.

For hints, go to: https://www.geogebra.org/m/TeXKJjVc

21. Use the Triangle Midsegment theorem to show that the Centroid divides medians in the ratio of 2:1.
    For hints, go to: https://www.geogebra.org/m/ScZJ3mqC

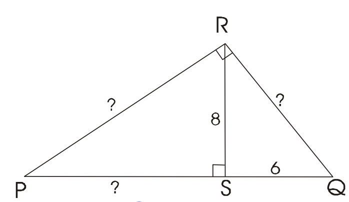


22. Three midsegments in a triangle split the triangle into 4 smaller triangles. Show that they all are congruent.
23. DE is a midsegment of the triangle ABC, point F is randomly selected on the side AC. Show that G is the midpoint of BF.

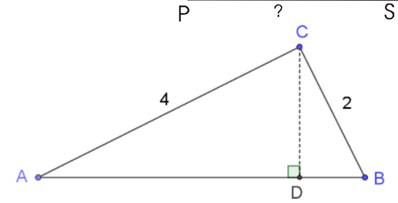


Right Triangle Theorems

24. Formulate and prove Euclid's theorems about the height and legs in a right triangle. https://www.geogebra.org/m/tUYgVpRT
25. Use the previous result to prove the Pythagorean theorem.
26. The altitude to the hypotenuse of a right triangle is the geometric mean of the segments into which the altitude divides the hypotenuse. Try to make sense of what the theorem is saying. Draw a picture to explain it. Prove the theorem.
27. Use triangle similarity to solve for the missing dimensions. Don't forget to justify why the triangles you chose are similar. https://www.geogebra.org/m/W3MTSFDc
28. Find the missing numbers. (If your answer is not a whole number, express it as a fraction or leave it in a radical form.)



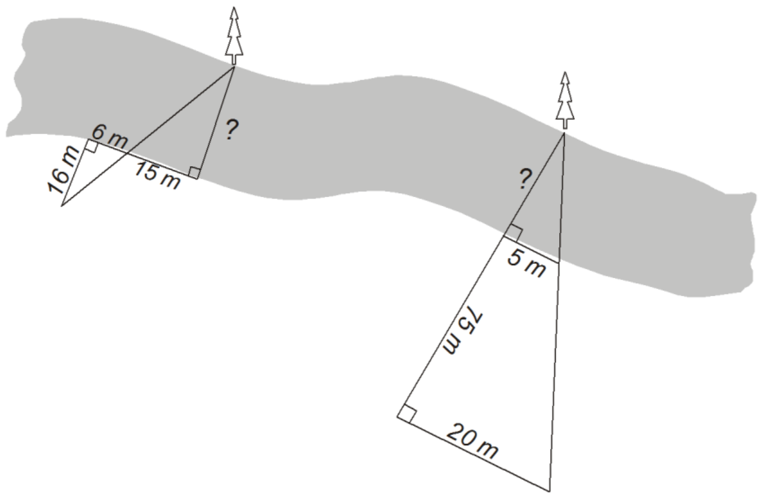
29. Right triangle has legs 2 and 4 units long. Calculate its height falling on the hypotenuse. (Hint: Use the fact that all triangles in the picture are similar. The answer after simplification is $\frac{4}{5}\sqrt{5}$)



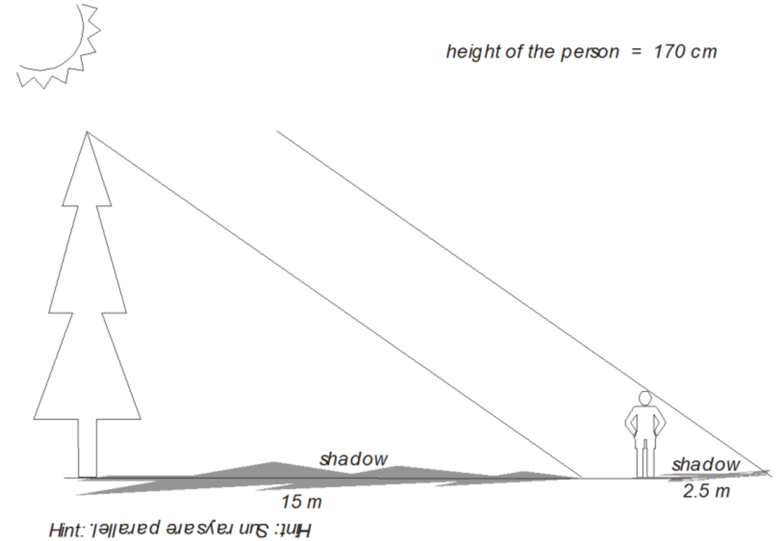
30. The pictures on the next page show five methods of finding inaccessible dimensions (indirect measurement). Picture 1 shows two methods of finding the width of a river and all remaining show finding the height of a tree.

    a. Briefly explain the method ("what would you do" to find the dimensions).
    b. Identify appropriate similar triangles (of course, with justification).
    c. Calculate the missing dimensions.

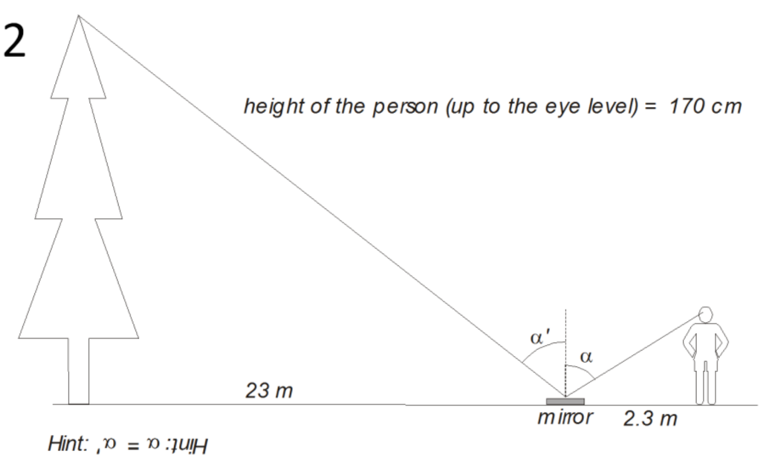
1


16 m
6 m
15 m
?
?
5 m
75 m
20 m

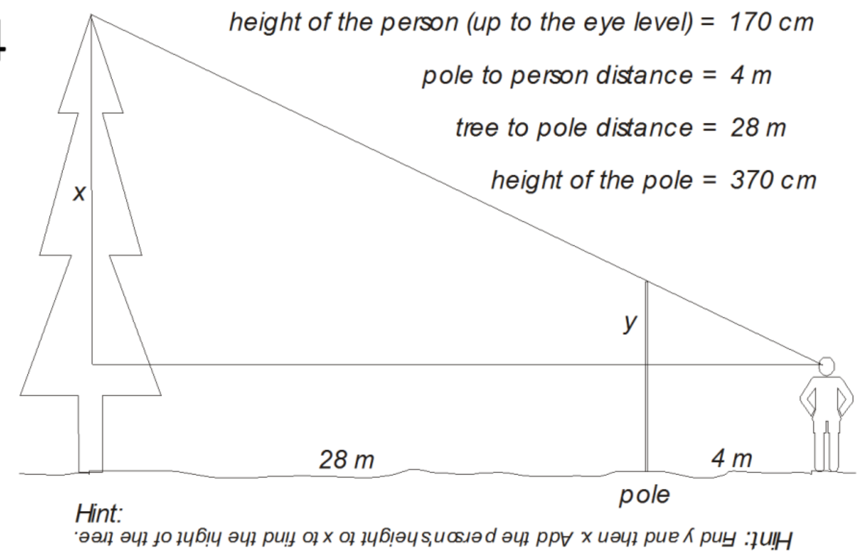
3


height of the person = 170 cm
shadow
15 m
shadow
2.5 m
Hint: Sun rays are parallel.

2


height of the person (up to the eye level) = 170 cm
α'
α
23 m
mirror
2.3 m
Hint: α = α'

4


height of the person (up to the eye level) = 170 cm
pole to person distance = 4 m
tree to pole distance = 28 m
height of the pole = 370 cm
x
y
28 m
4 m
pole
Hint:
Hint: Find y and then x. Add the person's height to x to find the hight of the tree.

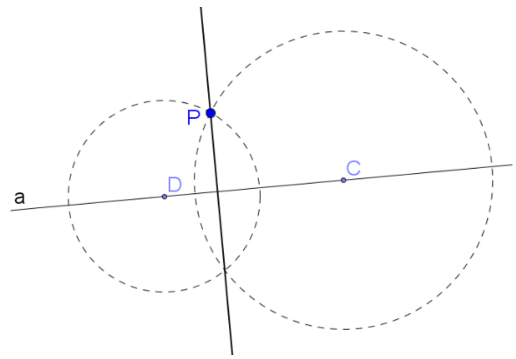
Circles

31. *Review the vocabulary*: center, radius, diameter, chord, tangent, secant, minor and major arc, central angle, inscribed angle, to subtend, to intercept, Thales' circle.

32. Explain (prove) why a tangent is always perpendicular to the radius at the point of tangency.

33. Explain (prove) why the center of a circle always lies on the perpendicular bisector of any of the circle's chords.

34. If two circles intersect at two points, the common chord is perpendicular to the line connecting the centers of the two circles. Prove it.



35. Draw a picture representing a relationship between inscribed and central angles corresponding to the same arc. What is the relationship? Prove it. (The complete proof has 3 parts).

36. State the Thales's Theorem and prove it.

37. What is the set of all points on the plane from which you can see a given line segment at 90° angle? Explain.

38. Show the set of all points in the plane from which you can see a given line segment at a 40° angle. Explain how to construct such a set and why your construction works (prove it). (Hints: https://www.geogebra.org/m/mrGDmPKt)

    a. Discuss the construction of the set of all points from which you can see a given line segment at 140° angle. Can you use the very same construction as in the previous example to show the set?

39. Find the center of a given circle by using the following methods. Explain why your methods work.

    i. Technology (GeoGebra).
    ii. Tools other than compass and straightedge.

40. Draw a circle. Without a protractor, draw an arc that subtends the following inscribed angles. (Think of corresponding central angles – can you draw them without a protractor?).

    a. 45° b. 30° c. 15°

Theorems about angles formed by chords, secants and tangents.

Prove the following circle theorems (Hints are provided in the GGB book: https://www.geogebra.org/m/gku3fgmx):

41. Angle of two chords:
    If two chords intersect, then the measure of any one of the vertical angles they form is equal to half the sum of the measures of the two arcs intercepted by the two vertical angles.

42. Angle of two secants:
    If two secants intersect outside a circle, the measure of the acute angle formed is half the difference of the measures of the intercepted arcs.
43. Angle of chord and tangent:
    The measure of an angle formed when a chord intersects a tangent line at the point of tangency is half the measure of the arc intercepted by the chord and the tangent line.
44. Angle of secant and tangent:
    If a secant and tangent lines intersect outside a circle, the measure of the angle formed is half the difference of the measure of the larger intercepted arc and the smaller intercepted arc.
45. Angle of two tangents:
    If two tangent lines intersect outside a circle, the measure of the angle formed is half the difference of the measure of the larger intercepted arc and the smaller intercepted arc.
46. Angles in cyclic quadrilateral. Cyclic quadrilateral is a quadrilateral with all vertices on a single circle. What can you say about the opposite angles in a cyclic quadrilateral? Prove your conclusion.