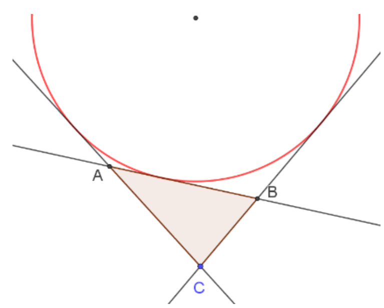
General comments. The exam will take place on MS Teams. Any construction that you need or want to submit will have to be saved as a ggb or tns file and uploaded to Blackboard. You will also take a picture of your finished work and upload it to Bb. I will be happy to answer technical questions during the exam. As to your proof write-up, it does not have to meet the "textbook" or "journal" standards but it should be coherent and consistent. To ensure that, always re-read your proof and ask yourself:

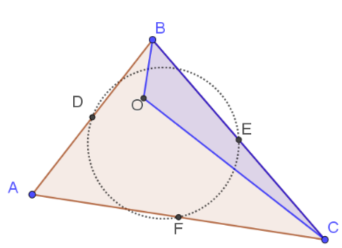
- Is each claim (statement) you wrote supported with a clear justification?
- Are there any leaps in reasoning? (Assumptions or statements you are considering without mentioning them).
- Does your conclusion match exactly what you wanted to prove?

More triangle geometry

1. You should be able to construct all 4 triangle centers (Centroid, Orthocenter, Circumcenter, Incenter).
2. You should be able to construct Circumscribed and Inscribed circles to a given triangle (keep in mind that incircle construction has an extra step).
3. Referring to the properties of the perpendicular bisector, explain why the circumcenter is constructed as an intersection of perpendicular bisectors. (Hint: "equidistant" might be a good key word to use)
4. Referring to the properties of the angle bisector, explain why the incenter is constructed as an intersection of angle bisectors. (Hint: "equidistant" might be a good key word to use)
5. Excircles. Given a triangle ABC, construct a circle tangent to its one side (AB) and the extensions of two other sides (AC, BC, as in the picture).

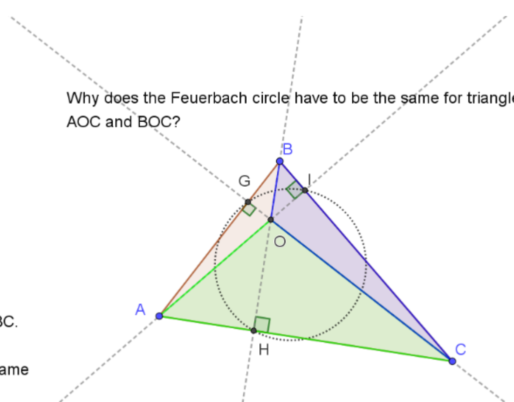


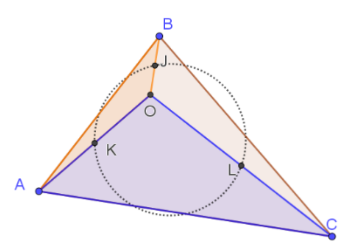
6. Describe or define the Feuerbach circle.
7. Triangle ABC has the orthocenter O. What is the orthocenter of the triangle ABO? Justify it. (You do not have to strictly adhere to the two-column proof format, just make sure you provide valid arguments, and clear reasoning without gaps in it.
8. In the following problems, O is the orthocenter of the triangle ABC. Explain why the given triangles must have the same Feuerbach circle:
   a. ∆ABC and ∆BOC (picture on the left)
   b. ∆AOC and ∆BOC (in the middle)
   c. ∆AOC and ∆AOB (on the right).



D, E, F are the midpoints of the sides AB, BC and AC respectively. O is the orthocenter of ∆ABC.

Why does the Feuerbach circle have to be the same for triangles ABC and BOC?





J, K, L are the midpoints of the segments BO, AO and CO respectively.
O is the orthocenter of ∆ABC..

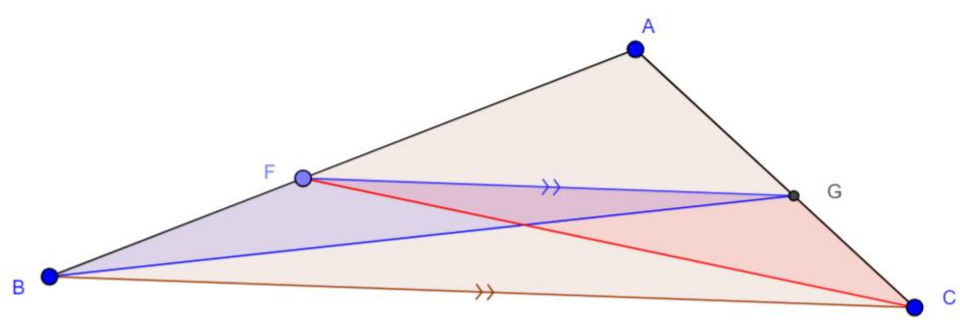
Why does the Feuerbach circle have to be the same for the triangles AOC and AOB?

Similarity and Triangle Similarity Conditions

9. When are two polygons similar?
   a. Does the definition of similar figures from the previous question hold for polygons?
   b. Does your definition of similar polygons work for other geometrical figures?
10. Define similar triangles.
11. List triangle similarity conditions.
12. Why no asa or aas similarity theorems are mentioned in geometry textbooks? Would they work?
13. Can we use the phrase “CPCTC” for similar triangles? How can the phrase be changed to reflect what’s important for similarity considerations?
14. Formulate and prove the Side-splitter theorem. https://www.geogebra.org/m/PdGTJvDC

Note: On the exam, I will not ask you to produce the whole proof, but I may ask questions regarding a particular step in the proof. For example a question can be phrased as follows:

In the proof of Side-splitter theorem, the picture below is used to justify a claim about triangles FGB and FGC. State the claim and justify (prove) it.



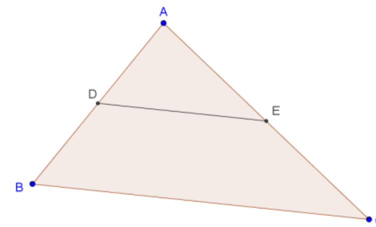
15. Formulate and prove corollary to the Side-splitter theorem.
16. Briefly discuss the importance of the Side-splitter theorem (and/or its corollary).
17. Prove the aa triangle similarity condition.
    https://www.geogebra.org/m/fzpjVwXK#material/WT6nJZ6V

Note: You don’t have to repeat the same reasoning in different parts of your proof. If some arguments in your proof are repeating, you will be allowed to use “shortcuts” on the exam. Such shortcuts are typically introduced by “Similarly, …”. For example, if you are asked to prove that all pairs of angles α,β, and γ are congruent your proof can have this structure:

i. Draw an appropriate picture and use full reasoning to prove that $\alpha = \beta$.
ii. Draw an appropriate picture that applies to β and γ but instead of repeating complete reasoning as in 1., simply state “Similarly (see picture 2), $\beta = \gamma$”
iii. Proceed with other steps or formulate your conclusion.

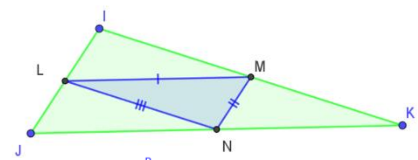
Triangle similarity applications

18. D is the midpoint of AB, E is the midpoint of AC. (The segment DE is called *midsegment*.).
    a. Identify similar triangles in the picture and provide proper justification.
    b. Formulate two observations about the midsegment (known as the midsegment theorem) and prove them.

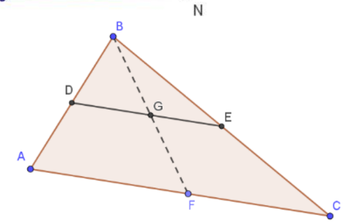


For hints, go to: https://www.geogebra.org/m/TeXKJjVc

19. Use the Triangle Midsegment theorem to show that the Centroid divides medians in the ratio of 2:1.
    For hints, go to: https://www.geogebra.org/m/ScZJ3mqC

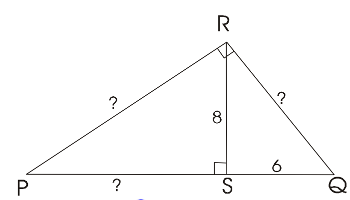


20. Three midsegments in a triangle split the triangle into 4 smaller triangles. Show that they all are congruent.
21. DE is a midsegment of the triangle ABC, point F is randomly selected on the side AC. Show that G is the midpoint of BF.

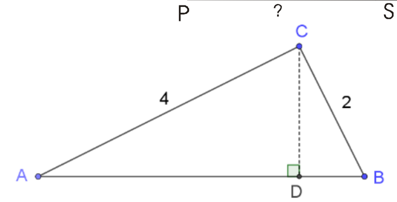


Right Triangle Theorems

22. Formulate and prove Euclid's theorems about the height and legs in a right triangle. https://www.geogebra.org/m/tUYgVpRT
23. Use the previous result to prove the Pythagorean theorem.
24. The altitude to the hypotenuse of a right triangle is the geometric mean of the segments into which the altitude divides the hypotenuse. Try to make sense of what the theorem is saying. Draw a picture to explain it. Prove the theorem. (Hint: Research what the geometric mean is if you are not familiar with it. Then rewrite the sentence using algebra to see if it looks familiar.)
25. Use triangle similarity to solve for the missing dimensions. Don't forget to justify why the triangles you chose are similar. https://www.geogebra.org/m/W3MTSFDc
26. Find the missing numbers (see the "?" mark in the picture). If your answer is not a whole number, express it as a fraction or leave it in a radical form.



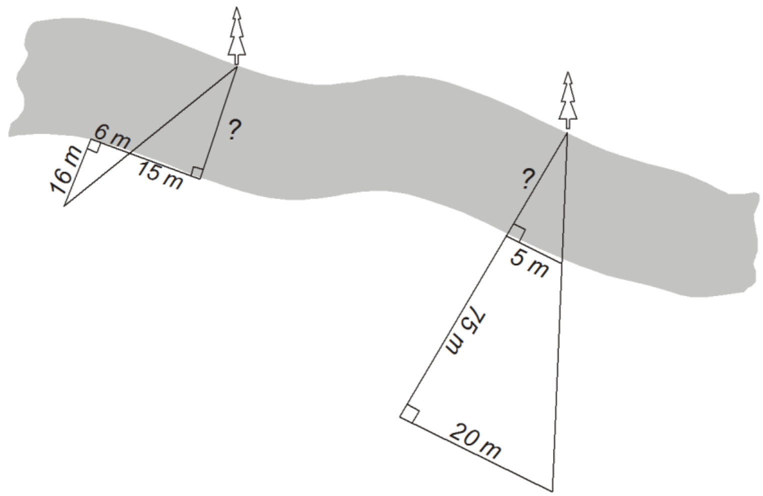
27. Right triangle has legs 2 and 4 units long. Calculate its height falling on the hypotenuse. (Hint: Use the fact that all triangles in the picture are similar. The answer after simplification is $\frac{4}{5}\sqrt{5}$)



28. The pictures on the next page show five methods of finding inaccessible dimensions (indirect measurement). Picture 1 shows two methods of finding the width of a river and all remaining show finding the height of a tree.

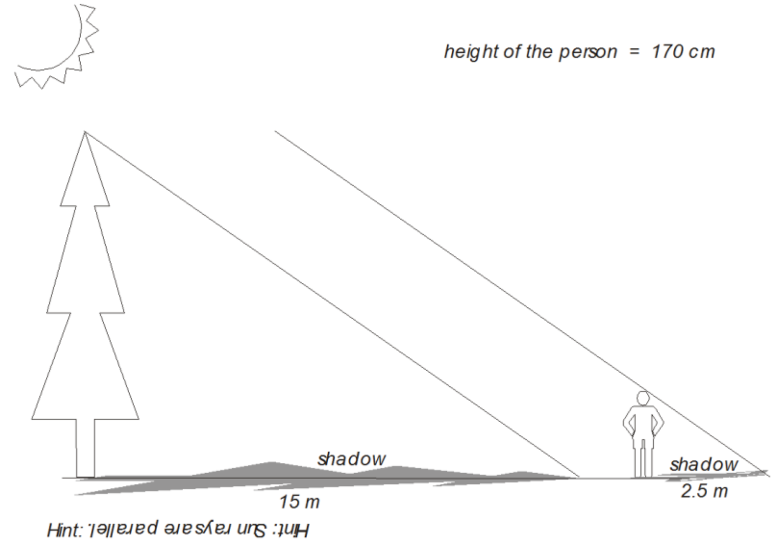
    a. Briefly explain the method ("what would you do" to find the dimensions).
    b. Identify appropriate similar triangles (of course, with justification).
    c. Calculate the missing dimensions.

1



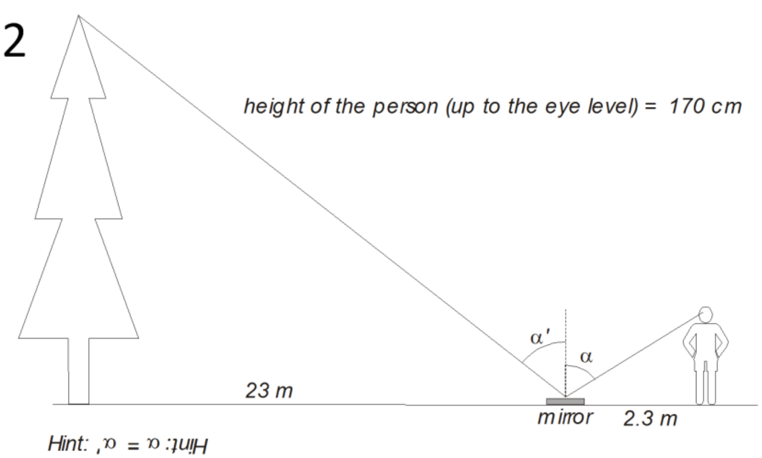
16 m
6 m
15 m
?
?
5 m
75 m
20 m

3



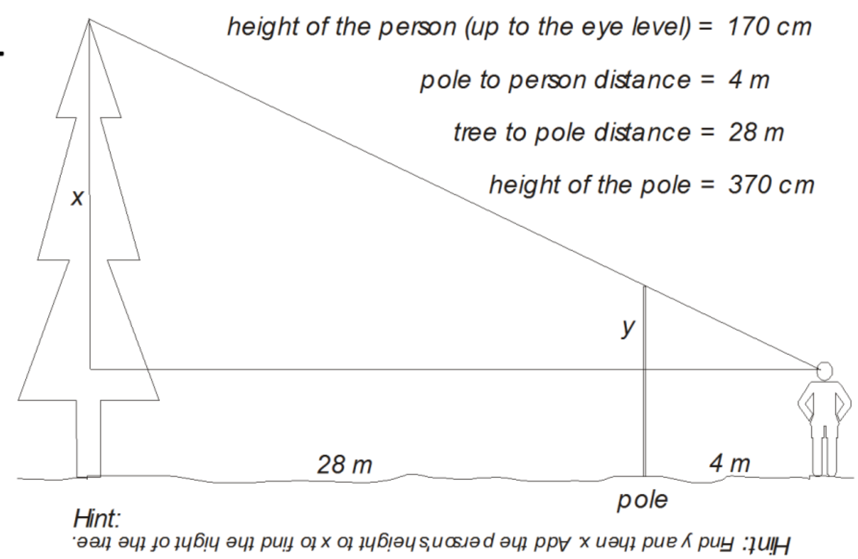
height of the person = 170 cm
shadow
shadow
15 m
2.5 m
Hint: Sun rays are parallel.

2



height of the person (up to the eye level) = 170 cm
α'
α
23 m
mirror
2.3 m
Hint: α = α'

4



height of the person (up to the eye level) = 170 cm
pole to person distance = 4 m
tree to pole distance = 28 m
height of the pole = 370 cm
x
y
28 m
4 m
pole
Hint:
Hint: Find y and then x. Add the person's height to x to find the hight of the tree.