Test #2 Study Guide (complete)

The exam will take place in the computer lab. You will work on a computer and submit (1) a test sheet with your answers (handwriting is fine) and (2) Geogebra files. You will submit the files via Blackboard (wait for a confirmation message). You are allowed to bring and use your own computer.

Similarity

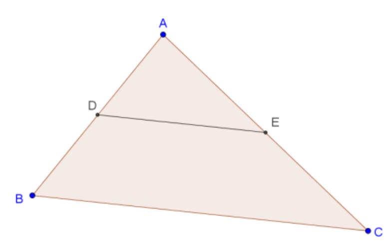
1. When are two figures (not necessarily polygons) similar?
   - Describe the concept on an intuitive level. How do students often describe similar figures?
   - Describe the concept mathematically.
2. When are two polygons similar?
   a. Does the definition of similar figures from the previous question hold for polygons?
   b. Does your definition of similar polygons work for other geometrical figures?

Triangle similarity conditions

3. Define similar triangles.
4. List triangle similarity conditions.
5. How are SAS and SSS congruence theorems same and different from sas and sss similarity theorems? (You don't need to state the theorems, just point to the similarities and differences).
6. Why no asa or aas similarity theorem is talked about in geometry textbooks? Would it work?
7. Can we use the phrase "CPCTC" for similar triangles? How can the phrase be changed to reflect what's important for similarity considerations?

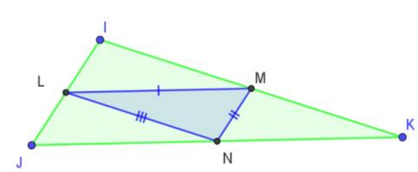
Triangle similarity applications

8. D is the midpoint of AB, E is the midpoint of AC. (The segment DE is called *midsegment*.).
   a. Identify similar triangles in the picture and provide proper justification.
   b. Formulate two observations about the midsegment (known as the midsegment theorem) and prove them.



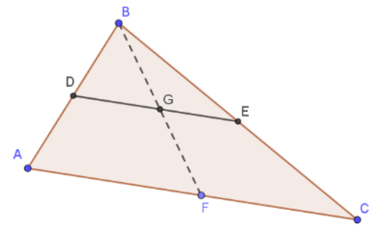
If you need hints, go to: https://www.geogebra.org/m/TeXKJjVc

9. Use the Triangle Midsegment theorem to show that the Centroid divides medians in the ratio of 2:1.
   If you need hints, go to: https://www.geogebra.org/m/ScZJ3mqC



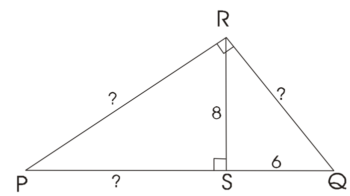
10. Three midsegments in a triangle split the triangle into 4 smaller triangles. Show that they all are congruent.

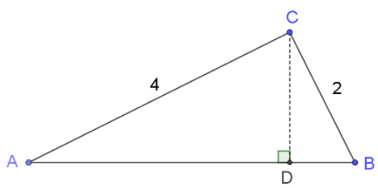
11. DE is a midsegment of the triangle ABC, point F is randomly selected on the side AC. Show that G is the midpoint of BF.



Right Triangle Theorems

12. Formulate and prove Euclid's theorems about the height and legs in a right triangle. https://www.geogebra.org/m/tUYgVpRT
13. Use the previous result to prove the Pythagorean theorem.
14. Use triangle similarity to solve for the missing dimensions. Don't forget to justify why the triangles you chose are similar. https://www.geogebra.org/m/W3MTSFDc
15. Find the missing numbers. (If your answer is not a whole number, express it as a fraction or leave it in a radical form.) Picture on the right >>>>





16. Right triangle has legs 2 and 4 units long. Calculate its height falling on the hypotenuse. (Hint: Use the fact that all triangles in the picture are similar. The answer after simplification is $\frac{4}{5}\sqrt{5}$)
17. The Midsegment Quadrilateral. Construct a general quadrilateral ABCD. Find midpoints of all its sides WXYZ. Connect these points to form the midsegment quadrilateral.
Use the triangle midsegment theorem to find and explain:

    a. For any quadrilateral, what is its midsegment quadrilateral?
    b. If WXYZ is a rectangle, what kind of quadrilateral is ABCD?
    c. The perimeter of WXYZ in terms of the lengths of diagonals AC and BD.

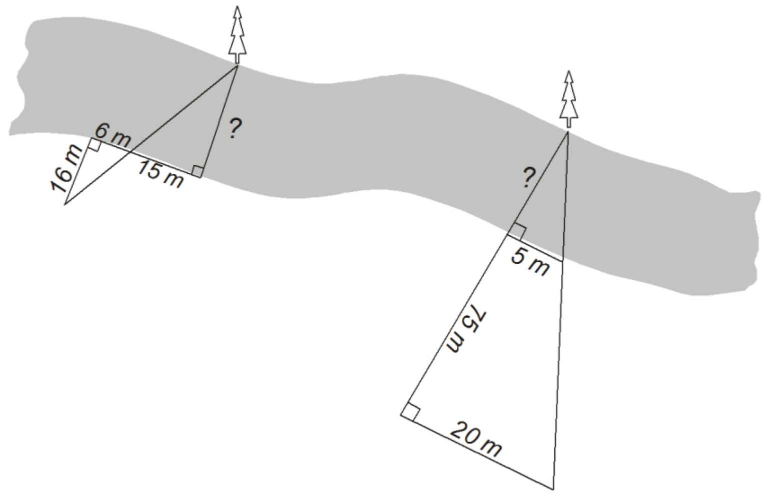
18. Play with your construction and finish the sentence: Midsegment quadrilateral is a square if the original quadrilateral is (chose one).
    i. Parallelogram
    ii. Square
    iii. Rhombus

    Explain your answer by showing why the other two options are incorrect.

19. The pictures on the next page show five methods of finding inaccessible dimensions (indirect measurement). Picture 1 shows two methods of finding the width of a river and all remaining show finding the height of a tree.

    a. Briefly explain the method ("what would you do" to find the dimensions).
    b. Identify appropriate similar triangles (of course, with justification).
    c. Calculate the missing dimensions.

1



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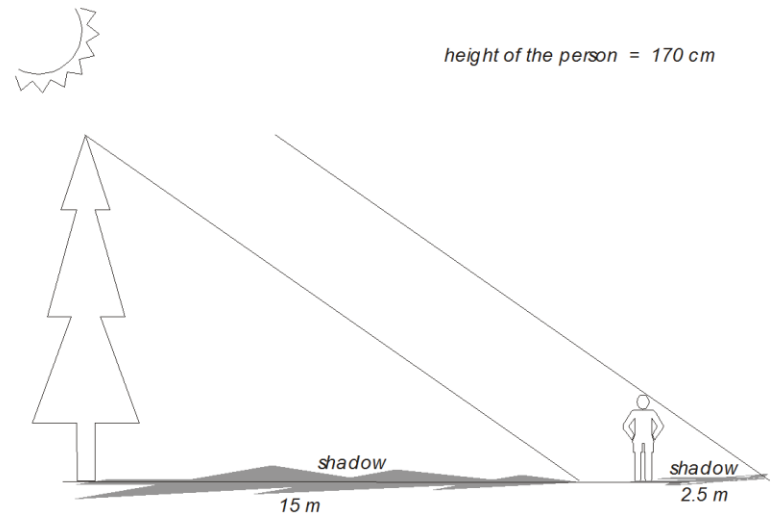
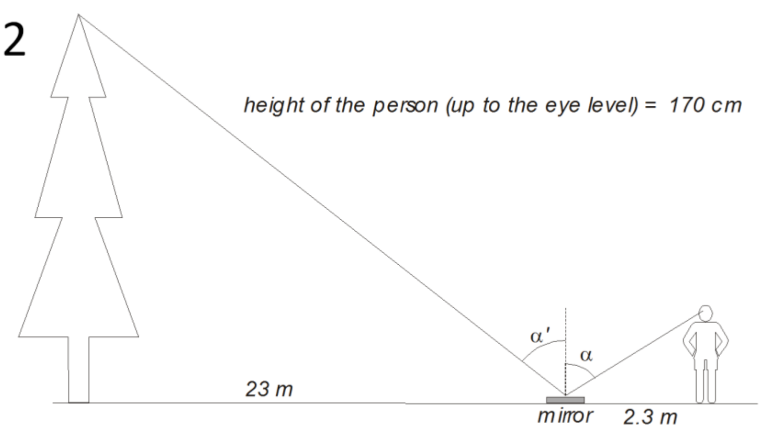


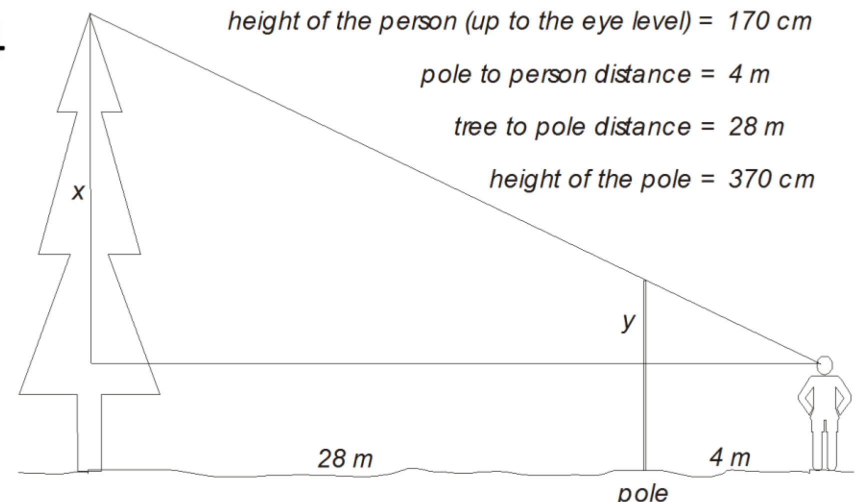
Hint: Sun rays are parallel.

2



Hint: α = α'

4



Hint:
Hint: Find y and then x. Add the person's height to x to find the hight of the tree.

Quadrilaterals

Construction of quadrilaterals

You should be able to construct quadrilaterals (kite, trapezoid, parallelogram, rhombus, rectangle, square) given their minimal definitions (you do not have to memorize the definitions, they will be provided). Make sure your object is not over- or under- constrained.

In the above activity, you do not have to strictly adhere to our minimal definitions. However, you should be able to see how minimal definitions govern the construction of quadrilaterals and construct some quadrilaterals from their minimal definitions:

1. Construct a kite as
    a. A quadrilateral with two distinct pairs of congruent adjacent sides.
    b. A quadrilateral that is symmetrical with respect to one of its diagonals.
    c. A quadrilateral with two congruent adjacent sides and one diagonal being an angle bisector of the angle formed by these two congruent sides.
2. Construct a rhombus as
    a. A quadrilateral with four congruent sides.
    b. A parallelogram with two congruent adjacent sides.
3. Construct a rectangle as
    a. A quadrilateral with 3 right angles.
    b. A parallelogram with one right angle.

Properties of Quadrilaterals

Justifications for the properties of quadrilaterals are based on triangle congruence and many such problems were solved in the chapter on triangle congruence. Here are more examples of what you should be able to prove. Try to prove them first without using hints.

4. Square: a quadrilateral with four congruent sides and one right angle. Show (prove) that this definition implies that all angles have to be right angles. (Hint: https://www.geogebra.org/m/jqnbkrqz. Keep in mind that only the definition of a square is given so only approach #1 is applicable. You can use approach 2 only if you first show that square is a parallelogram.)
5. Rhombus: a quadrilateral with four congruent sides. Show (prove) that rhombus is a parallelogram. (Hint: https://www.geogebra.org/m/xm93wvn7)
6. Isosceles trapezoid: a trapezoid with one pair of congruent base angles. Show (prove) that the legs of isosceles trapezoid (the two sides that are not parallel) are congruent. (Hint: https://www.geogebra.org/m/fndwu98w).
7. Parallelogram: a quadrilateral with two pairs of parallel sides. Show (prove) that opposite sides of parallelogram are congruent. (Hint: https://www.geogebra.org/m/bcue5ngg)
8. Trapezoid: a quadrilateral with at least one pair of parallel sides. Draw a trapezoid with its diagonals. These diagonals partition the trapezoid into 4 triangles. Two of them are similar and two of them have the same area. Show which ones are similar and which ones have the same area and provide mathematical justification of your claim (Hint: https://www.geogebra.org/m/rpmmsmrb)

Circles

1. *Review the vocabulary*: center, radius, diameter, chord, tangent, secant, minor and major arc, central angle, inscribed angle.

2. Draw a circle c and a point T on it. How do you construct a tangent to c touching it at the point T? Explain why your construction works (what theorem are you using?)

3. Explain (prove) why the center of a circle always lies on the perpendicular bisector of any of the circle's chords.

4. If two circles intersect at two points, the common chord is perpendicular to the line connecting the centers of the two circles. Draw a picture and prove the theorem.

5. Draw a picture representing a relationship between inscribed and central angles corresponding to the same arc. What is the relationship? Prove it. (The complete proof has 3 parts).

6. State the Thales's Theorem and prove it.

7. Demonstrate one practical application of the Thales's theorem.

8. In Geogebra, explain at least two different methods of finding unknown center of a circle and explain why your methods work (prove them). If your methods do not involve the Thales's theorem, perform a third construction that is based on the theorem.

Theorems about angles formed by chords, secants and tangents.

9. Angle of two chords:
   If two chords intersect, then the measure of any one of the vertical angles they form is equal to half the sum of the measures of the two arcs intercepted by the two vertical angles. For hints, see https://www.geogebra.org/m/hm2pntwn .
10. Angle of two secants:
   If two secants intersect outside a circle, the measure of the acute angle formed is half the difference of the measures of the intercepted arcs. Hints:
   https://www.geogebra.org/m/g798ucwq .