

Test #1 Study Guide

The exam will take place in the computer lab. You will work on a computer and submit (1) a test sheet with your answers (handwriting is fine) and (2) your Geogebra file(s). You will submit the files through Blackboard (wait for a confirmation that I received your file(s)). You are allowed to bring and use your own computer. While taking exam, only Geogebra can be open (and CMU Blackboard when you are sending your file(s)).

*Please note: You are fully responsible for testing that everything works as you expect on the computer you are going to use. This includes saving files on your desktop or hard drive that you will upload to Blackboard. I will willingly assist you **before** the exam day to resolve technical issues.*

Deductive System of Geometry

1. Who was Euclid and what is he famous for?
2. Definitions are an important part of geometry. Although you do not have to memorize definitions word by word, you should be able to tell what makes a good definition. What issues might arise from the following students' definitions and how would you react to them? (Note that some of them may be correct)
 - An angle is where two lines intersect.
 - An angle is the number of degrees at the intersection of two lines or rays.
 - Acute triangle is a triangle with an acute angle.
 - Obtuse triangle is a triangle with only two acute angles.
 - Polygon is a 2D figure composed of line segments.
 - Polygon is a line drawn without lifting a pencil.
3. Review the angles vocabulary. Describe the context in which we use these terms.
 - a) Vertical, Corresponding, Alternate Interior, Alternate Exterior,... (when two lines are cut by a transversal). You should be able to reason about the sizes of these angles if given two parallel lines cut by a transversal.
 - b) Acute, Right, Obtuse, Straight, Reflex angle... (reflects the angle size)
 - c) Supplementary, complementary (reflects the sum of two angles' measures)
 - d) Opposite, adjacent (reflects the position in a polygon)
4. What is a postulate? On how many postulates did Euclid build his Geometry?
5. What is a theorem? What are the roles of proof?
6. Prove the following theorems. Write their proofs in a two-column format.
 - a) "Vertical Angles Theorem".
 - b) If two lines a , b are parallel and line c is perpendicular to a , then c is perpendicular also to b .

Constructions

7. Carry out the following constructions using methods given in *italics*. Briefly record the steps. You can try the constructions in Geogebra applets: <https://www.geogebra.org/m/GUDwUySb>.

Paper folding, compass & ruler, GeoGebra:

- Midpoint

- Perpendicular Bisector of a line segment
- Angle Bisector
- Perpendicular Line (to a given line through a given point on the given line)
- Perpendicular Line (to a given line through a given point not on the given line)
- Parallel Line (to a given line through a given point not on the given line)

Compass & Ruler and GeoGebra:

- Transferring an Angle

Polygons

8. *Definition* – How do we define polygons?
9. *Vocabulary*: Side, Vertex, Diagonal, Interior angle, Exterior angle; Regular polygon
10. *Convexity and concavity* of figures and polygons – what is a convex/non-convex figure? How would you define a convex shape using mathematical terms?
11. *Classification of polygons*. Review the names of polygons (up to a 10-sided polygon). What is a regular polygon?
12. *Interior angles sum*.
 - Derive a formula for the sum of interior angles of an n-sided polygon using two different methods.
 - Derive a formula for the interior angle of an n-sided regular polygon.
 - Find the interior angle of a regular pentagon, octagon, ... etc.
13. *Exterior angles sum*.
 - Derive the formula for the exterior angles sum of an n-sided polygon using
 - Visual “car method”
 - The interior angle sum formula and algebra
14. Equilateral means “equal sides”. Equiangular means “equal angles”. Decide if the following statements are true or false. You do not have to prove it if it is true, but if it is false, give/draw a counterexample.
 - A regular polygon is always equilateral.
 - An equiangular polygon is also equilateral.
 - Pentagon that has all sides congruent is a regular pentagon.

Triangles

15. *Triangle vocabulary*:
 - Vertex, Side, Interior Angle, Exterior Angle,
 - Altitude, Median, Centroid, Orthocenter, Incenter, Circumcenter
 - Equilateral, Equiangular, Isosceles, Scalene, Acute, Obtuse, Right triangle.

- Base, Vertex angle, Base angle, Hypotenuse, Leg (which triangles do these terms refer to?)

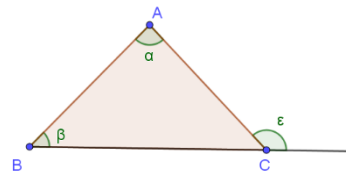
16. Are the following definitions and statements complete/correct? If not, suggest how to fix them.

- Acute triangle is a triangle with an acute angle.
- Height is a line segment perpendicular to a side of a triangle.
- Median connects midpoints of two sides of a triangle.
- Equilateral triangle is also an equiangular one.
- Equilateral triangle is also an isosceles one.
- Isosceles triangle is also an equiangular one.

Etc.

Angles in a Triangle

17. Formulate and prove the “Interior Angle Sum in a Triangle” Theorem. (See your notes from the beginning of the semester).
18. How would you design a simple activity for elementary school students in which they could discover the theorem? (Using paper, scissors or other manipulatives)
19. “The Exterior Angle in a Triangle Theorem” states: The size of an exterior angle at a vertex of a triangle (ϵ) equals the sum of the sizes of the interior angles at the other two vertices of the triangle (α and β). Make sense of the theorem. Prove it.
20. The isosceles triangle theorem talks about the base angles of an isosceles triangle. State the theorem and prove it

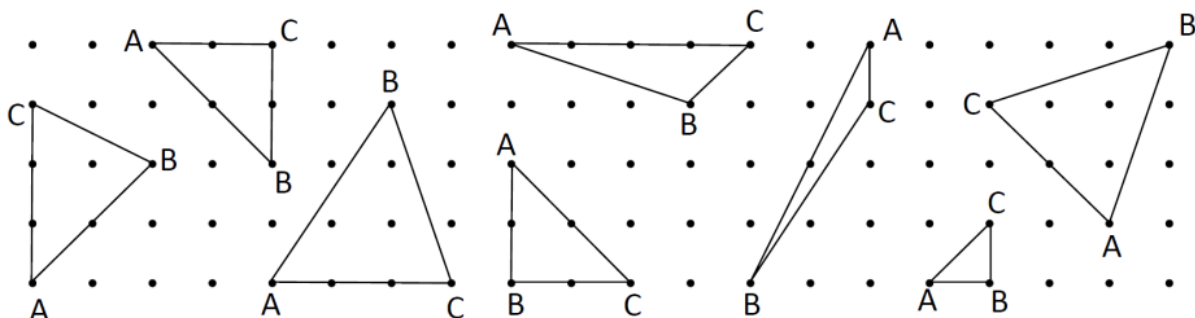


Triangle Inequality

21. Is it always possible to construct a triangle given 3 sides? Explain.
22. Design an elementary or middle school activity, in which the students would discover the Triangle inequality theorem (or criterion).
23. Decide if the following triples can represent triangle sides. If so, specify if the triangle is an isosceles, equilateral, scalene, Can you also tell if it is a right triangle without drawing it?

- a) 3,7,9
- b) 15, 10, 5
- c) 12, 8, 12
- d) 12, 15, 10
- e) 15, 8, 8
- f) 6, 23, 16

24. Draw **all three** heights of the given triangles. Calculate their areas.



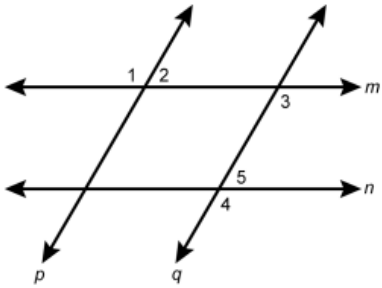
25. Which of the triangle centers (Centroid, Orthocenter, Incenter and Circumcenter) can fall outside of the triangle? When does it happen?
26. Given a triangle, construct its
 - a) Circumscribed circle
 - b) Inscribed circle.

Triangle Congruence

27. How do we define triangle congruence? (Note that the question is asking for the definition. This is different from stating SSS, ASA, ... theorems).
28. Decide if the following is sufficient to say that two triangles are congruent. If they are, state a congruence theorem that applies. If they are not necessarily congruent, give a clear counterexample.
 - a) Both triangles have a side 3 cm long, another side 2 cm long and an angle 45°
 - b) Both triangles have a 30° angle, 3 cm side and 5 cm side.
 - c) Both triangles have three 60° angles.
 - d) Both triangles have a 6cm side, 30° angle and 50° angle.
 - e) Both triangles have a 3cm side, 4cm side and 5cm side.
 - f) Both triangles have two 45° angles and the side adjacent to both angles is 6 cm long.
 - g) Both triangles have two 7cm sides and an angle adjacent to both sides is 70° .
 - h) Both triangles have a 3 cm side, 5 cm side and the angle opposite to the 5cm side is 30° .
 - i) Both triangles have a 3 cm side, 5 cm side and the angle opposite to the 3cm side is 30° .
29. What's the difference between ASA and AAS congruence theorems?
30. Knowing that ASA holds, prove the AAS theorem.
31. Can we use "SSA" theorem under certain circumstances? Describe the circumstances.
32. Identify pairs of congruent triangles: <https://www.geogebra.org/m/s53r7rus>
33. Use triangle congruence to prove some of the properties of quadrilaterals. Images are provided, you do not have to learn definitions of quadrilaterals (at this time).
<https://www.geogebra.org/m/exev8kmw>

The following page contains 3 questions in a MTTC format (practice problems).

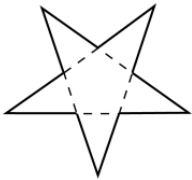
Use the diagram below to answer the question that follows.



Given that lines m and n are parallel, which of the following conditions guarantees that line p is parallel to line q ?

- A. $\angle 2$ and $\angle 5$ are complementary
- B. $\angle 3$ is congruent to $\angle 4$
- C. $\angle 1$ and $\angle 5$ are supplementary
- D. $\angle 2$ is congruent to $\angle 4$

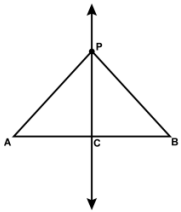
Use the diagram below to answer the question that follows.



A 5-pointed star is constructed by extending the sides of a regular pentagon until they intersect, as shown in the diagram. What is the sum of the angles in the points of the star?

- A. 90°
- B. 180°
- C. 360°
- D. 540°

Use the diagram below to answer the question that follows.



A student who is trying to prove that any point on the perpendicular bisector of a line segment is equidistant from the endpoints of that segment draws this diagram. An intermediate step in the proof will be $\triangle APC \cong \triangle BPC$. Which of the following triangle congruence theorems can be used to justify this statement?

- A. SSS
- B. AAS
- C. ASA
- D. SAS