Problem solving: Steps and Strategies

Problem Solving means engaging in a task for which the solution method is not known in advance. (NCTM's Standards 2000). In this sense, a *problem* is an opposite to *routine* in which solution method is known from the very beginning.

A *problem* is a relative concept: *Add 6 and 7* is obviously a routine task for us but it may become a problem for a first grader.

What does it take to succeed in solving a routine task? Follow all steps of your procedure and make sure you won't make a mistake. *Concentration* is the main factor.

What does it take to succeed in problem solving? Well, the answer is much more complex. **To be a successful problem solver**, you:

1. MUST BE ACTIVE and PERSISTENT. Guess, check, suggest, test, experiment – draw pictures, set up equations, try to organize information in tables, etc. If you get stuck, DON'T take it as a signal to give up. Take it as a signal to change your point of view, try another strategy or organize information differently.

2. MUST HAVE SOME EXPERIENCE. The more experience with problem solving you have, the better are your odds to solve your next problem. Good "intuition" and "luck" go hand in hand with experience. Don't avoid problems, they help you to become a better problem solver.

3. MUST HAVE SOME KNOW-HOW ("master your craft"). In math, you can set up a perfect equation to solve your problem but it is of a little use unless you know how to solve it.

The following page summarizes steps in problem solving. It comes from G. Polya and is based on experience of many successful problem solvers. By no means it is an *algorithm*. You cannot follow it step by step and expect to solve your problem. It is meant to help you to "get off the mark". Anytime you get stuck, try to go over the questions on the sheet. They may help you to find a different perspective.

Step 2: *Devising a plan* is a very important step and contains different problem solving ***strategies***. A mathematics or elementary teacher must have a solid arsenal of problem solving strategies. This part of the course should give you opportunity to experience some of them.

Sources:

Process Standards of NCTM's Principles and Standards for School Mathematics 2000.
G. Polya, "How to Solve It", 2nd ed., Princeton University Press, 1957
Hecht, T., Sklenáriková, Z.: Methods of Solving Math Problems. (In Slovak only)

Four steps of problem solving by Polya:

1. Understand the problem

- Do you understand all words?
- What are you asked to find out or show?
- Can you restate the problem in your own words?
- What is unknown? Which are unknowns?
- List all information the problem provides. Is any information missing. Is any information redundant?
- Can you draw a picture or diagram to help you understand the problem?
- Can you work out some numerical examples that would help make the problem more clear?

2. Devise a plan (Problem Solving Strategies)

- *Guess and check*
- *Look for a system*
- *Solve a simpler problem or a special case of the original problem.*
- *Solve a related problem.*
- *Make an organized list. List all possibilities.*
- *Draw a picture or diagram (tree diagram, Venn diagrams, …)*
- *Look for a pattern*
- *Work backwards*
- *Make a table*
- *Use deduction*
- *Use a variable. Set up an equation.*
- *Experiment, Act it out*

3. Carry out the plan

- Be patient – most problems are not solved quickly nor on the first attempt
- Allow yourself a reasonable amount of time to solve the problem. If you are not successful, ask for help or put the problem aside for a while. Inspiration comes even when you do not expect it.
- If a plan does not work immediately, be persistent
- Do not let yourself get discouraged by starting over several times. A new beginning often brings a better strategy and increased odds to solve the problem.
- If a strategy you chose does not work out, don't forget to change your point of view and try another strategy.

4. Look back (reflect)

- Does your answer make sense? Did you answer all of the questions?
- What did you learn by doing this?
- Could you have done this problem another way – using a different strategy?
- Can you see a simpler solution?
- Can you generalize your solution? Can you make it a routine?