This study guide provides sample problems that can be included in the exam. It makes references to the course pack and problems we solved in class. Make sure to use both the course pack and your notes when preparing for the exam.

Pattern recognition – Sequences and Figurative numbers

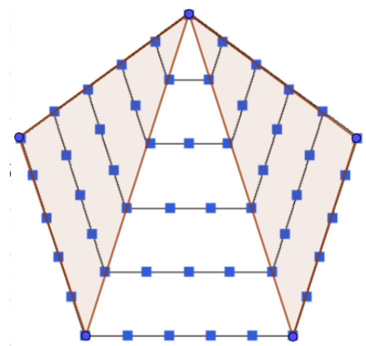
1. Continue the pattern (next 3 terms) and find the formula for the n-th term:

   - 10, 14, 18, 22, 26, …
   - 2, 6, 12, 20, 30, 42, …
   - -12, 2, 48, 144, 308, ...

   Other examples: Course pack, pages 6-8

2. Formulas for problems in #1 can be derived by observing differences between the terms, differences between these differences, etc. Explain why this does not work for geometric sequences. Write the formula for the n-th term of a geometric sequence.

3. Describe triangular numbers and derive the formula for the n-th triangular number.
4. Describe square numbers and derive the formula for the n-th square number.
5. Describe pentagonal numbers and derive the formula for the n-th pentagonal number.



6. Two copies of a triangular number can be arranged in a rectangle. Show it and use this fact to derive the formula for the n-th triangular number.

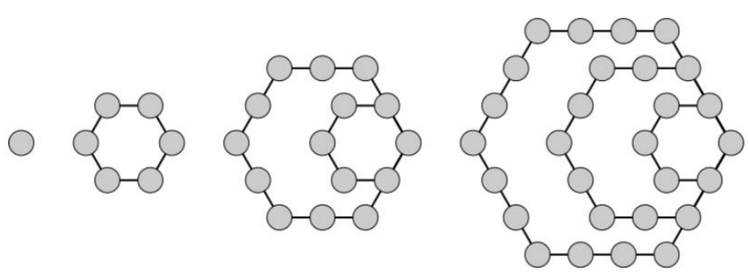
7. Use the picture on the right to derive the formula for the n-th pentagonal number.

8. Formula for the N-th pentagonal number can also be derived from the following two equations. Derive it (algebraically) and draw a picture that justifies each approach.

   Method 1: $P_n = T_n + 2T_{n-1}$

   Method 2: $P_n = S_n + T_{n-1}$

9. The following picture shows a sequence of first four hexagonal numbers. Write two more hexagonal numbers and derive the formula for the n-th hexagonal number.



10. Formula for the N-th hexagonal number can also be derived from the following four equation. Derive it (algebraically) and draw a picture that justifies each approach.

    Method 1: $H_n = 2T_n + T_{n-1} + T_{n-2} - 1$

    Method 2: $H_n = S_n + S_{n-1} + (n-1)$

    Method 3: $H_n = S_n + 2T_{n-1}$

    Method 4: $H_n = 2S_n - n$

Pascal’s triangle patterns

1. Show a “hockey stick” pattern and briefly justify it.

2. Formulate a probability problem that can be easily solved from the Pascal’s triangle. Solve it.

3. Calculate a few powers of 11 and explain their connection to the Pascal’s triangle.

4. Identify triangular numbers in the Pascal’s triangle. Explain why the line of numbers next to the triangular numbers is a cubic sequence.

5. Describe how binomial expansion relates to the Pascal’s triangle and expand $(t - u)^8$

6. Observe the pattern below. Describe and explain it.
   (If you said that two numbers in the first row added together (4+6) give the number below them (10), try to find yet another pattern given by these three numbers. Notice the shaded row of numbers – it only works if one of the numbers is from this row)