Fractals

1. Describe in your own words what a fractal is.
2. Draw a (smaller version of) Sierpinski Triangle on the isometric dot paper.
3. Assume the area of the stage 1 (original) of the Sierpinski Triangle is 1. Find the area of the n^{th} stage of the Sierpinski Triangle.
4. Assume the perimeter of the stage 1 (original) of the Sierpinski triangle is 1. Find the perimeter of the n^{th} stage of the Sierpinski Triangle.
5. What is the area and perimeter of the Sierpinski triangle if we allow n to be infinitely large?
6. Use the previous result to explain the concept of divergent and convergent geometric series. When will such series converge?
7. Draw a (smaller version of) Koch Snowflake on the isometric dot paper.
8. Assume the perimeter of the stage 1 (original) of the Koch Snowflake is 1. Find the perimeter of the n^{th} stage of the Sierpinski Triangle.
9. What is the perimeter of Koch Snowflake if we allow n to be infinitely large?
10. Although you don't have to calculate the area of Koch Snowflake, will it be a finite number or will it increase without any boundary if we allow n to be infinitely large?
11. Use your answers in 5, 9, and 10 to describe one property that makes the fractals very special geometrical objects.

Infinity

12. Someone says that there are twice as many natural numbers as there are even numbers. How would you respond?
13. Someone says that a line segment with the length of 10 cm contains more points than a segment only 1 cm long. How would you respond?
14. What is a cardinal number and how is it different from the number of elements?
15. What cardinal numbers are you familiar with?
16. Can a proper subset of a set be equivalent to the set itself?
17. Which sets have the cardinal number equal to $\aleph_0$? What do we call such sets?
18. Is ∞ a number? Is ∞ a cardinal number? Explain.
19. What can you say about sets with the same cardinal number?
20. What is the largest cardinal number?
21. Describe a set with a cardinal number larger than c.
22. Can there be the set of all sets?
23. True of false (and of course, explain your answer).

    - If A and B are infinite sets, then they are equivalent.
    - If set A is infinite and set B can be put in 1-to-1 correspondence with a proper subset of A, then B is an infinite set.
    - There is infinitely many natural numbers and real numbers, therefore there is no more real numbers than there are natural ones.

24. Explain the nature of the Zeno's paradox of the Tortoise and Achilles. What was Zeno maintaining? How can we use current tools of mathematics to show that Achilles will eventually catch up with the tortoise? Pick a specific example (for example the tortoise has a 1 mile head start and is running ½ the speed of Achilles) and show mathematically or graphically when Achilles will catch up with the tortoise.
25. Here's another Zeno's paradox that claims that motion is impossible. You cannot travel one meter until after you have first gone a half meter. But you cannot go a half meter until after you have first gone a quarter meter, etc. You would have to take infinitely many "steps" to travel 1 meter, which is impossible. Explain where this reasoning goes wrong.

Ghost whisperer

26. Think of a two digit numbers, then subtract the sum of its digits. What is special about the resulting number? Explain mathematically why it happens.
27. Will this work for any natural number (not necessarily two digit?).
28. Explain why the following gives the original number as the result: Think of a number. Add 8, multiply the result by 4, then subtract 3, add 7, divide by 4 and subtract 9.
29. Invent your own "guess my number" game with at least 4 steps and explain how to quickly find the original number.

Cryptography

30. The following are coding and decoding matrices (in this order): $\begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}$, $\begin{bmatrix} -1 & 0 \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$
    - Decode the following message: -19, 86, -18, 63, -14, 49, -27, 60, -21, 78, -2, 85
    - Code a message of your choice so that it can be decoded with the decoding matrix.