Final Exam Review

Math 2300 - Section 880

Instructions. Be sure to show your work and explain your reasoning for full credit.

1. Solve the following integrals.

   (a) $\displaystyle\int \frac{\ln(x)}{x}\,dx$

   (b) $\displaystyle\int_0^1 e^x \cos(2x)\,dx$

   (c) $\displaystyle\int \sinh(x)\sin(x)\,dx$

   (d) $\displaystyle\int_0^{\pi/2} \sin^4(x)\,dx$

   (e) $\displaystyle\int \sin^3(x)\cos^3(x)\,dx$

   (f) $\displaystyle\int_0^2 \frac{x^2}{\sqrt{x^2+4}}\,dx$

   (g) $\displaystyle\int \frac{x^4+1}{x^3+9x}\,dx$

   (h) $\displaystyle\int_{-\pi/2}^{\pi/2} \sin(2x)\cos(3x)\,dx$

2. (a) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{x^2+1}$

   (b) Determine whether $\displaystyle\int_0^{\infty} \frac{dx}{\sqrt{x+x^3}}$ converges or diverges.

   (c) Determine whether $\displaystyle\int_2^{\infty} \frac{dx}{\ln(x)}$ converges or diverges.

3. (a) Find the area bounded by the curves $y = x^2 - 4$ and $y = 3x$.

   (b) Sketch the polar curve $r^2 = \cos(2\theta)$ (or equivalently, $r = \pm\sqrt{\cos(2\theta)}$). This is called the **lemniscate**.

   (c) Find the area contained in the right leaf of the lemniscate in the previous problem.

4. (a) Find the volume of the solid of revolution obtained by rotating the region bounded by $y = \sqrt{x}$ and $y = x^2$ about the line $y = -1$.

   (b) Find the volume of the solid of revolution obtained by rotating the region in the first quadrant bounded by $y = \sqrt{x}$ and the line $y = 1$ about the y-axis.

5. (a) Find the arc-length of the curve given by the graph of $f(x) = 2e^x + \frac{1}{8}e^{-x}$ with $x \in [0, \ln(2)]$.

   (b) Find the arc-length of the polar curve given by $r = 1 + \cos(\theta)$.

6. Suppose a force of 10 units is required to stretch a spring 0.1 units from its equilibrium position. How much work is needed to stretch the spring 0.25 units from its equilibrium position?

7. Find the centroid of the region in the first quadrant bounded by the graph of $x^2 + 4$ and the line $x = 2$.

8. Determine whether the following sequences and series converge or diverge. Justify your answers.

   (a) $\left\{ \frac{1}{n} \sin\left(\frac{1}{n}\right) \right\}$

   (b) $\sum_{n=0}^{\infty} n^2$

   (c) $\sum_{n=0}^{\infty} \frac{1}{n^2+2}$

   (d) $\sum_{n=0}^{\infty} \frac{1}{n^2-2}$

   (e) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$

   (f) $\sum_{n=0}^{\infty} \frac{\cos(\pi n)|\cos(n)|}{n}$

9. Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{n^n}$.

10. Determine what the following series are equal to.

    (a) $\sum_{n=0}^{\infty} \frac{2}{4^n}$

    (b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$

    (c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$

    (d) $\sum_{n=0}^{\infty} \frac{n}{3^n}$

11. Find the 5^{th} Taylor polynomial about 0 of $\tan(2x)$.

12. Find the Taylor series about 0 of $\sinh(x)$.

13. Find the Taylor series about 0 of $\frac{\sin(x)}{x}$.

14. Given the following IVP, use Euler's method to approximate $y(3/2)$ using a step-size of $h = 1/2$.

$$y' = x - y, \quad y(0) = 1$$

15. Find the general solution to $y' = \sec(y)\cos(t)$.

16. Given a radioactive material with a half-life of 1200 years, how long does it take for an initial sample of 100g to decay to 10g?

17. Assume that a population of rabbits exhibits logistic growth. If initially the population is 100, and the carrying capacity of the population is 1000, find the population after 10 years if after 1 year, the total population is 150.

18. A fish tank has 150 litres of water with 20 grams of salt dissolved in it. The salt concentration needs to be increased to 1 gram per litre. A solution with concentration of 3 grams of salt per litre is poured into the tank at a rate of 2 litres per minute, after which the contents of the tank is stirred. At the same time, the tank is drained at the same rate. How long will it take to increase the concentration to the needed specification?

19. Use a series solution to solve $y'' + xy = x$

20. Convert the following from polar coordinates to Cartesian coordinates.

    (a) $(r, \theta) = (1, \pi/3)$
    (b) $(r, \theta) = (3, -5\pi/6)$
    (c) $(r, \theta) = (-1, \pi)$
    (d) $(r, \theta) = (-2, -4\pi/3)$

21. Convert the following from Cartesian coordinates to polar coordinates.

    (a) $(x, y) = (1, 2)$
    (b) $(x, y) = (-3, 4)$
    (c) $(x, y) = (4, -3)$
    (d) $(x, y) = (-5, -2)$.

22. Parametrise the tangent line to the curve $c(t) = (\sin(t), t^2, \cos(3t))$ at $c(0)$.

23. Find all of the angles θ where the tangent line to $r = 1 + \cos^2(\theta)$ is horizontal or vertical.