Homework #4 (Continued)

Math 2300 - Section 880

Due: Thursday, Sep 17

Instructions. Be sure to show your work and explain your reasoning for full credit. Be aware that this homework assignment also has problems from the textbook (as indicated on the course website).

NAME Solutions

1. Find the derivative of $f(x) = \operatorname{arcosh}(x)$. (**Hint**: use implicit differentiation, or first find a formula for $\operatorname{arcosh}(x)$ in terms of elementary functions.)

Method 1 (easier):

$y = \operatorname{arcosh}(x) \Rightarrow x = \cosh y$

$\Rightarrow 1 = \sinh(y) \frac{dy}{dx}$

$\Rightarrow \frac{1}{\sinh(y)} = \frac{dy}{dx}$

$\Rightarrow \frac{1}{\sqrt{x^2 - 1}} = \frac{dy}{dx}$

$\cosh^2 y - \sinh^2 y = 1$

$\Rightarrow \sinh y = \sqrt{\cosh^2 y - 1}$

Method 2 (harder):

$y = \operatorname{arcosh}(x) \Rightarrow x = \cosh y = \frac{1}{2}(e^y + e^{-y})$

$\Rightarrow e^y - 2x + e^{-y} = 0$

$\Rightarrow e^{2y} - 2xe^y + 1 = 0$

$\Rightarrow e^y = \frac{2x \pm \sqrt{4x^2 - 4}}{2} = x \pm \sqrt{x^2 - 1}$

$= x + \sqrt{x^2 - 1}$ (*)

$\Rightarrow \boxed{y = \ln\left(x + \sqrt{x^2 - 1}\right)}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x + \sqrt{x^2-1}}\left(1 + \frac{x}{\sqrt{x^2-1}}\right) = \frac{1}{\sqrt{x^2 - 1}}$

(*) Careful: $\cosh y$ is not invertible for $y \in (-\infty, \infty)$, so restrict to where it is invertible: $y \in [0, \infty)$ $\Rightarrow e^y \geq 1 \Rightarrow$ choose the "+" solution!

2. Solve the following integral using a hyperbolic trigonometric substitution:

$$\int \frac{dx}{\sqrt{x^2+1}}.$$

SOL'N: Since $\cosh^2 t = 1 + \sinh^2 t$, let $x = \sinh t$.

$\Rightarrow dx = \cosh t \, dt$

So, $\displaystyle\int \frac{dx}{\sqrt{x^2+1}} = \int \frac{\cosh t \, dt}{\cosh t} = \int dt = t + C$

$= \operatorname{arsinh}(x) + C$

(or, $= \ln\left(x + \sqrt{x^2+1}\right) + C$)