

## Homework #7 (Continued)

Math 2300 - Section 880

Due: Thursday, Oct 8

**Instructions.** Be sure to show your work and explain your reasoning for full credit. Be aware that this homework assignment also has problems from the textbook (as indicated on the course website).

NAME Solutions

**Warning:** This homework assignment has two pages.

1. (a) Show that the following sequence converges, and find its limit.

$$\lim_{n \rightarrow \infty} \frac{n+1}{n} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) = 1$$

$$\text{So, } \lim_{n \rightarrow \infty} \ln\left(\frac{n+1}{n}\right) = \ln\left(\lim_{n \rightarrow \infty} \frac{n+1}{n}\right) = \ln(1) = 0$$

- (b) Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$$

$$\ln\left(\frac{n+1}{n}\right) = \ln(n+1) - \ln n \quad \text{So,}$$

$$\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right) = \ln 2 - \ln 1 + \ln 3 - \ln 2 + \ln 4 - \ln 3 + \dots$$

This is a telescoping sum!

$$= \lim_{n \rightarrow \infty} (\ln n) - \ln 1 = \infty. \quad \text{So, the series diverges.}$$

2. Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^n$$

We know  $\lim_{n \rightarrow \infty} \left(\frac{n+1}{n}\right)^n = e$ . So, by the divergence test, the series diverges (since  $e \neq 0$ ).