Taylor Polynomial homework problems

1. Do but don't turn in: memorize the formula for the nth-degree Taylor Polynomial for $f(x)$ centered at $x = a$:

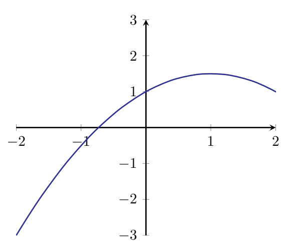
$$T_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + ... + \frac{f^{(n)}(a)}{n!}(x-a)^n$$
$$= \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i$$

2. Find the 4th degree Taylor polynomial for $\tan x$ centered at $x = 0$.

3. The function $f(x)$ is approximated near $x = 0$ by the 3rd degree Taylor polynomial $T_3(x) = 4 - 3x + \frac{x^2}{5} + 4x^3$. Give the values of $f(0)$, $f'(0)$, $f''(0)$ and $f'''(0)$.

4. Find the 10th degree Taylor polynomial centered at $x = 1$ of the function $f(x) = 2x^2 - x + 1$.

5. Here's a graph of $f(x)$:



If the 2nd-degree Taylor polynomial centered at $a = 0$ for $f(x)$ is $T_2(x) = ax^2 + bx + c$, determine the signs of a, b and c.

6. Show your work in an organized way.
   (a) Find the 7th degree Taylor polynomial centered at $a = 0$ for $\sin(x)$.
   (b) Use $T_7(x)$ to estimate $\sin(3°)$. Don't forget to convert to radians.
   (c) Compare your answer to the estimate for $\sin(3°)$ given by your calculator or other technology. How accurate were you?

7. This problem asks for Taylor polynomials for $f(x) = \ln(1 + x)$ centered at $a = 0$. Show your work in an organized way.
   (a) Find the 4th, 5th and 6th degree Taylor polynomials for $f(x)$ centered $a = 0$.
   (b) Find the nth degree Taylor polynomial for $f(x)$ centered $a = 0$, written in expanded form.
   (c) Find the nth degree Taylor polynomial for $f(x)$ centered $a = 0$, written in summation notation.
   (d) Use the 7th degree Taylor polynomial to estimate $\ln(2)$.
   (e) Compare your answer to the estimate for $\ln(2)$ given by your calculator. How accurate were you?

(f) Looking at the Taylor polynomials, explain why this estimate is less accurate than the estimate in the previous problem for $\sin(3°)$.

8. Do, but don't turn in: memorize the nth degree Taylor polynomials centered at $a = 0$ for e^x, $\sin(x)$, $\cos(x)$, $\ln(1 + x)$ and $\dfrac{1}{1 - x}$. Be able to write each of them down with ease in both expanded form and sigma-notation.