MATH 2300: Honors Calculus II, Fall 2014
FINAL EXAM

Thursday, December 18, 2014

YOUR NAME:

Important note: SHOW ALL WORK. BOX YOUR ANSWERS. Calculators are not allowed.
No books, notes, etc.

Problem	Points	Score
1	8	
2	6	
3	8	
4	12	
5	10	
6	9	
7	7	
8	8	
9	16	
10	7	
11	9	
TOTAL	100	

"On my honor, as a University of Colorado at Boulder student, I have neither given nor received
unauthorized assistance on this work."

SIGNATURE:

(1) (4 points each) Compute

(a) $\dfrac{\partial}{\partial x}\left(xe^{\sqrt{xy}}\right)$

(b) $f_y(1, \pi)$ if $f(x, y) = 4x^2y + e^x + y\sin(xy)$

(2) (6 points) Find the Taylor series of order 3 for $g(x) = \int e^{x^2}\, dx$ near $x = 0$.

(3) (8 points) Find the center of mass of the region bounded by $y = 3x^2$ and $y = 3$ with density $\delta = 2$.

(4) (2 points each) Do the following converge or diverge? **CIRCLE** your answer, no work necessary.

a) $\sum_{n=1}^{\infty} \frac{(n+1)^2}{2n+3}$ CONVERGE   DIVERGE

b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ CONVERGE   DIVERGE

c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n+1}$ CONVERGE   DIVERGE

d) $\sum_{n=1}^{\infty} \frac{5n+2}{n^2-7n}$ CONVERGE   DIVERGE

e) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ CONVERGE   DIVERGE

f) $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^n$ CONVERGE   DIVERGE

(5) (a) (3 points) Write the Taylor series for $\sin(x)$ near $x = 0$.

(b) (5 points) What is the minimum degree Taylor polynomial of $\sin x$ near $x = 0$ necessary to approximate $\sin(0.1)$ to within 0.0002?

(c) (2 points) Using the degree you found in part (b), approximate $\sin(0.1)$ to within 0.0002.

(6) (9 points) Find the area inside $r = 2\sin(2\theta)$ and outside $r = 1$.

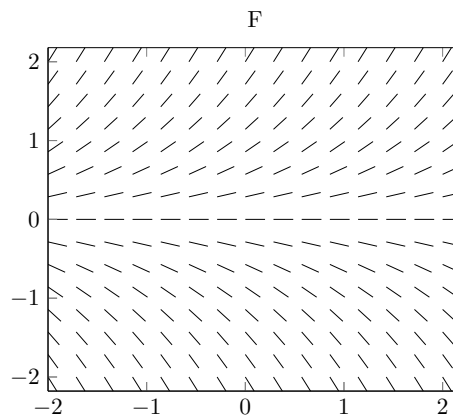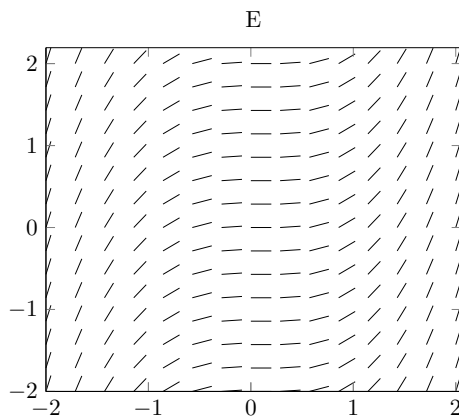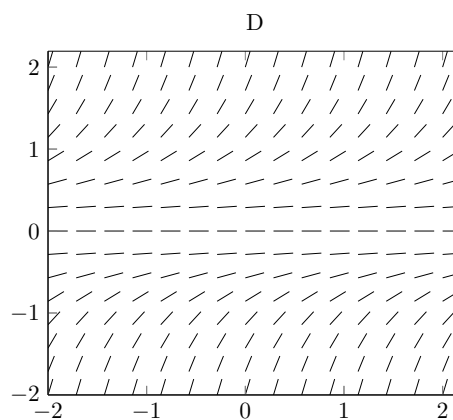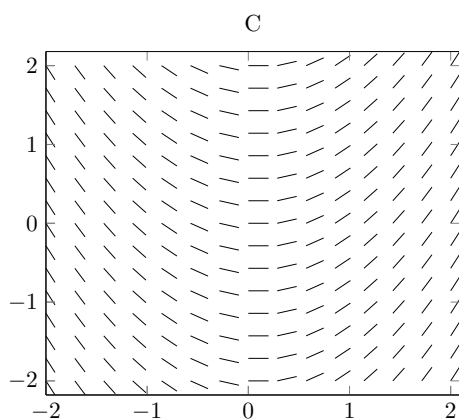(7) (7 points) Find the general solution to the differential equation $\dfrac{dy}{dx} = \dfrac{(y^2 + 1)}{x^3}$.
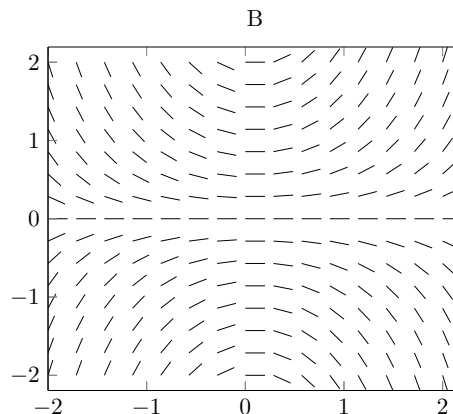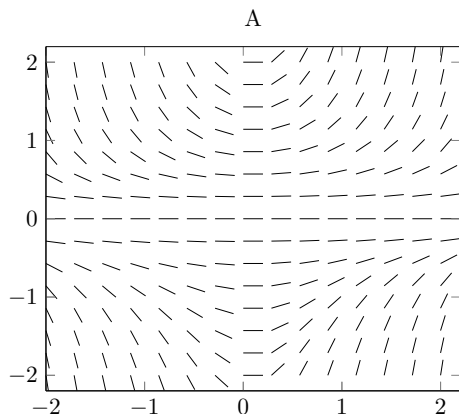
(8) (2 points each) Write the letter of the corresponding slope field under each differential equation.

$$\frac{dy}{dx} = xy^2, \qquad \frac{dy}{dx} = x^2, \qquad \frac{dy}{dx} = y, \qquad \frac{dy}{dx} = x$$

A

B

C

D

E

F

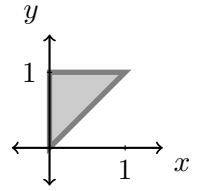(9) (4 points each) Compute the following indefinite integrals.

(a) $\int \dfrac{\cos(\sqrt{y})}{\sqrt{y}}\, dy$

(b) $\int \dfrac{1}{x^2 - 1}\, dx$

(c) $\int \dfrac{x+1}{\sqrt{x}}\, dx$

(d) $\int \cosh(2x)e^x\, dx$

(10) (7 points) Integrate e^{y^2} over the region shown below.

(11) (a) (3 points) Write the Taylor series of $\cos(x)$ near $x = 0$.

(b) (6 points) Does $\displaystyle\int_0^1 \frac{\cos(x)}{x}\,dx$ converge or diverge? Show your work.