

Quiz #3

Math 3430 - Section 002

Instructions. Be sure to show your work and explain your reasoning for full credit.

NAME Solutions

There are 2 problems on this quiz. Please turn the paper over to see the second question.

1. Solve the following initial value problem for $t > 0$.

$$t^2 \frac{d^2 y}{dt^2} - 3t \frac{dy}{dt} + 4y = 0; \quad y(1) = 1, \quad y'(1) = 0.$$

Set $y(t) = t^r$. Get $t^2 r(r-1)t^{r-2} - 3trt^{r-1} + 4t^r = 0$

$$\Rightarrow r^2 - 4r + 4 = 0$$

$$\Rightarrow (r-2)^2 = 0 \Rightarrow r=2 \text{ (repeated root)}$$

General solution: $y(t) = c_1 t^2 + c_2 t^2 \ln t$

$$y(1) = c_1 = 1$$

$$y'(t) = 2t + 2c_2 t \ln t + c_2 t$$

$$y'(1) = 2 + c_2 = 0 \Rightarrow c_2 = -2$$

Solution: $y(t) = t^2 - 2t^2 \ln t$

Check: $y'(t) = 2t - 4t \ln t - 2t = -4t \ln t$

$$y''(t) = -4 \ln t - 4$$

$$t^2(-4 \ln t - 4) - 3t(-4t \ln t) + 4(t^2 - 2t^2 \ln t)$$

$$= (-4 + 12 - 8)t^2 \ln t + (-4 + 4)t^2 = 0 \quad \checkmark$$

2. (a) Find the Laplace transform of $e^{-4t} \sin(7t)$.

$$\mathcal{L}\{e^{-4t} \sin(7t)\} = ?$$

$$\mathcal{L}\{\sin(7t)\} = \frac{7}{s^2 + 49}$$

$$\text{So, } \mathcal{L}\{e^{-4t} \sin(7t)\} = \frac{7}{(s+4)^2 + 49}$$

(b) Find the inverse Laplace transform of $\frac{e^{-4s}}{s(s-1)}$.

$$\mathcal{L}^{-1}\left\{\frac{e^{-4s}}{s(s-1)}\right\} = ? \quad \mathcal{L}^{-1}\left\{\frac{1}{s(s-1)}\right\} = \int_0^t 1 \cdot e^u du = e^t - 1$$

$$\text{So, } \mathcal{L}^{-1}\left\{\frac{e^{-4s}}{s(s-1)}\right\} = H_4(t)(e^{t-4} - 1)$$

Alternatively: $\mathcal{L}^{-1}\left\{\frac{1}{s(s-1)}\right\} = \mathcal{L}^{-1}\left\{\frac{A}{s} + \frac{B}{s-1}\right\}$

$$\text{where } 1 = A(s-1) + Bs = (A+B)s - A \Rightarrow A = -1, B = 1$$

$$\text{So, } \mathcal{L}^{-1}\left\{\frac{-1}{s} + \frac{1}{s-1}\right\} = -1 + e^t, \text{ and so}$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-4s}}{s(s-1)}\right\} = H_4(t)(e^{t-4} - 1)$$