An Old Friend in New Clothes: 
Smith Charts in LabVIEW

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Abstract
The Smith Chart was developed by P. H. Smith of Bell Telephone Labs in 1939 to avoid tedious calculations involving complex numbers. It was used by engineers developing microwave systems in World War II. It has since become a widely used tool for the analysis and design of transmission line circuits and microwave circuit elements. Even after the advent of modern computers with powerful complex arithmetic tools, the Smith Chart continues to be utilized heavily. It has become such a standard that we continue to teach Smith Charts in our Electromagnetics classes and Microwave CAD packages commonly display their results on Smith Charts. NI LabVIEW, which is the software tool used in the Fields and Waves class at our institution, has recently made a Smith Chart utility available. This paper will investigate the incorporation of electronic Smith Charts in the instruction of transmission line techniques and provide examples that will marry the time-honored paper / pencil / protractor / compass tool with the state of the art computational tool.

Introduction
At our institution, electromagnetics (ELE 305: Fields & Waves, as it is formally called) is a junior-level course for electrical engineering students. ELE 305 is a one-semester, 4-credit-hour course described in more detail in 2008\cite{1}. LabVIEW is the software of choice in this class, as described in a 2007 article\cite{2}. The current textbook\cite{3} was chosen for its early transmission lines approach to motivate electrical engineering students for electromagnetics.

Figure 1: Smith Chart (courtesy of Walter Dvorak, used under the Creative Commons Attribution-Share Alike 3.0 Unported License).
One of the topics covered relatively early in the course is The Transmission Line Calculator, or more commonly known as the Smith Chart (Fig.1). It is a graphical tool which was developed by Philip H. Smith of Bell Telephone Labs to carry out complex-valued calculations associated with transmission line circuits as well as microwave circuit elements. Since the first reference to it in 1932, the Smith Chart was used extensively during the development of microwave systems in World War II and has gained wide acceptance in microwave equipment, transmission line and antenna design.

As we trace the history of the computer during the last 60 years, we might be tempted to discard the Smith Chart as a relic of the past. But the Smith Chart has withstood the test of time even though today’s computers are capable of carrying out computations with complex variables. Due to the industry’s familiarity with this paper (and pencil) tool, Microwave CAD packages continue to display their solutions on a Smith Chart. Therefore we continue to include this topic in the presentation of transmission line techniques. As described in a previous publication, LabVIEW is the software of choice in this course. Our discovery of the Smith Plot VI in a rather unlikely place among LabVIEW’s toolset sparked this investigation. This paper presents fundamentals of the Smith Chart and the programs we have written to demonstrate the use of LabVIEW’s Smith Chart VI as a term project. We will first provide descriptions of the terminology commonly used in the discussion of transmission lines. We will then describe the VIs we have developed, ending with an example.

**Background Information**

Any conductor whose length is comparable to the wavelength of the signal it is conducting can be considered a transmission line (or T-line for short). The most common example of a transmission line would be the coaxial cable through which most households receive their TV signals. All transmission lines have a characteristic impedance, $Z_0$ (casually referred to as the line impedance), which is determined by the properties of the materials used to construct it and its geometry. When a load (such as a resistor or the input circuitry of a TV receiver) is connected to the transmission line, the T-line is said to be terminated with the impedance of the load, $Z_L$. A voltage or current wave, upon reaching the load is reflected back into the transmission line. The reflection coefficient at the load, $\Gamma_L$, is determined by the characteristic impedance of the T-line, $Z_0$, and the load impedance $Z_L$ as described by equation (1):

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (1).$$

In general the reflection coefficient is a complex quantity. We note here that when $Z_L = Z_0$, that is, when the line impedance and load impedance match, the reflection coefficient is zero. Reflections in the line are undesirable and we try to match the load impedance to the line impedance. We cannot always guarantee that a given load’s impedance will match the characteristic impedance of a T-line. Therefore we need to introduce additional impedance (or admittance) so that the overall combination achieves an impedance match. One of these impedance matching techniques is to introduce an open ended or a shorted stub of transmission line in parallel to the existing line at an appropriate distance from the load. This is called shunt-stub matching and we have chosen this technique for our example.
In the case of an impedance mismatch, the incident wave and the reflected wave form a standing wave pattern. The ratio of the maximum amplitude of the voltage standing wave to its minimum is called the voltage standing wave ratio (VSWR):

$$VSWR = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

In the context of transmission lines, it is useful to normalize the load impedance to the line impedance. The normalized impedance will be designated by $z$:

$$z_L = \frac{z_L}{z_o}$$

The normalized impedance $z$ will have a real part $r$ (normalized resistance) and an imaginary part $x$ (normalized reactance). With this normalization, the reflection coefficient as defined by equation (1) becomes:

$$\Gamma_L = \frac{z_L - 1}{z_L + 1}$$

We can generalize the reflection coefficient as the ratio of the reflected wave to the incident wave at any point along the line and drop the subscript $L$. We note that this generalized reflection coefficient is a phasor,

$$\Gamma = \Gamma_L e^{-j\beta}$$

where $\beta$ is the wave number and $l$ is the distance toward the source from the load. Equation (4), defining the reflection coefficient in terms of the normalized load impedance can be reversed to describe the load impedance in terms of this generalized reflection coefficient $\Gamma$:

$$z = \frac{1 + \Gamma}{1 - \Gamma}$$

The Smith Chart performs this forward and reverse transformation between normalized impedance and reflection coefficient. If we superimpose the real and imaginary axes of the complex plane right in the center of the Smith Chart (Figure 2), then for any $\Gamma$ plotted on this plane, the Smith Chart computes the real and imaginary parts of the normalized impedance. Each circle on the Smith Chart is a contour of constant $r$. Each (curved) line is a contour of constant $x$. 
Figure 2: The complex plane superimposed on the Smith Chart (courtesy of Walter Dvorak, used under the Creative Commons Attribution-Share Alike 3.0 Unported License).

Implementation
LabVIEW’s Smith Plot VI accepts an array of points, where each point is a cluster containing the $r$ and $x$ values that make up the complex impedance, $z = r + jx$. That is the only task it accomplishes. Therefore, we developed a number of tools in the form of VIs to be able to demonstrate typical tasks accomplished with the Smith Chart. These tools and their descriptions are listed below.

**TL.vi** computes $\Gamma_L$ from $Z_L$ and $Z_o$ according to equation (4). Figure 3 show the block diagram for this VI.

![Figure 3: The Block Diagram of the subVI calculating the reflection coefficient ($\Gamma_L$).](image)

**VSWR.vi** computes VSWR from $\Gamma_L$ according to equation (2) (Figure 4).

![Figure 4: The block diagram for the VSWR subVI.](image)
**ydvii** computes, through an iterative process, the distance \( d \) where a shunt-stub is to be placed and outputs the admittance at that point (Figure 5).

![Figure 5: The Block Diagram of the \( y_d \) subVI.](image)

**yL.vi** computes the length of the shunt-stub and outputs the angles for the Smith Chart representation for these lengths (Figure 6).

![Figure 6: The Block Diagram for the shorted and open ended stub.](image)

**zin.vi** computes the input impedance of a transmission line of length \( l \) terminated with a load \( z_L \) using equation (7) (Figure 7):

\[
Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta \ell)}{Z_o + jZ_L \tan(\beta \ell)} \quad (7)
\]
Figure 7: The Block Diagram for the Input Impedance (Z_{in}) subVI.

Figure 8: The Front Panel for the final VI show all calculated values and the Smith Chart representation.
Example
To demonstrate how LabVIEW’s Smith Plot VI may be used, we have chosen Example 2.7 from our current textbook which states: “Construct the shorted shunt stub matching network for a 50-Ω line terminated in a load $Z_L = 20 - j55 \, \Omega$.” After going through the explanation of how this is done on the Smith Chart, the example finds the stub length to be $0.074 \lambda$. The result matches that of the example in the book. Figure 8 displays the results of our VI constructed for this purpose using the tools we developed.

![Figure 8: Results of our VI constructed using LabVIEW's Smith Chart tools.](image)

Conclusions and Recommendations for Future Work
LABVIEW’s digital Smith Chart may become very useful in efficiently teaching the use of the Smith Chart. However, it has a complicated and peculiar set of inputs which make its utilization complicated. What we have achieved in this project is the implementation of LabVIEW VIs so that a programmer can utilize the Smith Plot VI in the LabVIEW toolkit without going through tedious calculations: We have provided VIs which will convert reflection coefficient information to the input format for the Smith Plot VI. We have done the same for impedance information; given a characteristic impedance and a load impedance, we can calculate the normalized impedance and format the result for the input of the Smith Plot VI. We have written an iterative VI which can mimic going around the Smith Chart in a constant $|\Gamma|$ circle until a certain normalized resistance is reached and can output the corresponding length of wire travelled in terms of $\lambda$. In the example problem, we have utilized these tools to successfully match impedance by adding a stub. However, it only demonstrates the capabilities of LABVIEW’s Smith Chart in this specific application. There are other ways these tools can be utilized in the future. For example, we could calculate capacitors, inductors, quarter-wavelength...
stunt additions with these same tools. Other companies have already written software to accomplish these tasks. In 1992 Motorola produced MIMP (Motorola Impedance Matching Program) which can be used to produce an impedance matching circuit. It additionally shows how to implement it using the Smith Chart as shown in (Fig. 9).

This software and other programs like winSmith, LinSmith, and Smith32 enable engineers to specialize solely on the design of the system and not on the mechanics of using the Smith Chart. For more complex calculations than those mentioned in the textbook, these powerful programs help them in the analysis of each application. We have provided standardized tools to achieve the same capabilities of these stand-alone applications with LabVIEW. At the next offering of the course, we plan to utilize examples developed in LabVIEW and survey students about their effectiveness.

References