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Abstract

When a steel plate in an army ground vehicle is found to be defective, usually it is taken out of service for repairing without determining if the defect warrants withdrawal or not, whereas the defect might be minor and withdrawal wasteful. On the other hand the defect might be invisible, yet warranting replacement. In our work we investigate and establish a procedure for defect characterization – thin crack, defect size and shape, spall, etc. – so that a decision to withdraw may be thought-out, justifiable and ensure the safety of the vehicle and its passengers. The methodology at present examines the response of the hull under test to an excitatory signal from an eddy current probe. Knowing the response when there is no defect, if the response is different because of the defect, the test object is presently flagged as defective and the vehicle sent for repairs without assessing if the defect is serious enough for removal from service. In this paper, we extend that methodology to defect characterization. A defect shape is characterized by parametric dimensions and properties $\bar{h}$, and we then solve the associated finite element problem with the defect, optimizing $\bar{h}$ until the computations match the measurements. The methodology is demonstrated by reconstructing a crack in a vehicle hull plate using eddy current probes. The problem is shown to be amenable to rapid analysis using the new GPU capabilities.

Keywords: Electromagnetics, Finite Element Analysis, Optimization, Genetic Algorithm, NDE

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Introduction

The computational algorithms for device synthesis and NDE are often the same. In both we have a goal – a particular field configuration yielding the design performance in synthesis or to match exterior measurements in NDE. These techniques [1] are extended in this paper for estimating the size shape and location of defects in materials. An iterative approach is presented that repeatedly employs the finite element technique for modeling the forward problem to characterize the shape of defects in a steel plate.
We can calculate the magnetic field density for the known defects from the forward problem. But in our inverse problem we need to know the characteristics of the defect for that field configuration. In design optimization, the problem geometry is defined in terms of design parameters contained in a vector $\overline{h} = \{y_1, y_1, \ldots, y_n\}$ (Fig.1.). An objective function $F$ is defined as the sum of the squares of the difference between computed and desired performance values: at measurement points $i$,

$$F = \sum_i (B^i_{\text{calculated}} - B^i_{\text{Measured}})$$

(1)

By minimizing the objective function $F$ by any of the optimization methods, the characteristics of the defect can be estimated.

![Fig.1. Defect Model](image)

Zeroth order optimization is practicable in terms of avoiding the horrendous programming complexity of first order optimization although computations are extensive. The Genetic Algorithm (GA) which is a zeroth order optimization method is good at handling potentially huge search spaces. Therefore in our work we investigate how the GA can be used to estimate the defect characteristics. It is also inherently parallel so that it may be easily adapted to computations on the graphics processing unit [2]. In this paper, we evaluate the proposed algorithm by applying the GA to a numerical NDE problem.

**Problem Statement**

Magnetic fields in a ferromagnetic material can be generated by placing an AC (Alternative Current) coil on top of the material (Fig.1.). For direct current magnetization, the vector magnetic potential $A$, and the current density $J$, are related by

$$\nabla \times \frac{1}{\mu} \nabla \times A = J$$

(2)
where \( \mu \) is the spatial permeability distribution. The magnetic field density \( \mathbf{B} \) is related to the vector magnetic potential as,

\[
\nabla \times \mathbf{A} = \mathbf{B}
\]

(3)

In two dimensional problems, the current density \( \mathbf{J} \) and the magnetic vector potential \( \mathbf{A} \) will be in the \( \hat{z} \) direction (i.e. the transverse direction in which no changes occur). Therefore the magnetic field density \( \mathbf{B} \) will be in the \( \hat{x}, \hat{y} \) directions. From (2) and (3) we can write,

\[
\frac{\partial}{\partial x} \left( \frac{\partial A}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial A}{\partial y} \right) + J = 0
\]

(4)

Finite element analysis [3] provides solution to (4) by applying certain boundary conditions. This leads to the finite element matrix equation \([P][A] = [R]\) where \( P \) is the finite element stiffness matrix and \( R \) is a column vector.

**Methodology**

![Design Cycle for the computation process](image)

The computational process in inverse problem solution is shown in Fig.2. It requires solving for the vector of design parameters \( \mathbf{\bar{h}} \). We first generate the mesh from the latest parameter set \( \mathbf{\bar{h}} \). Mesh generation is a very important part of finite element analysis based optimization. We need to use a parameter based mesh generator. As optimization proceeds, in each iteration, the
parameters are changed. For optimization to go on non-stop, the mesh needs to be generated for the new parameters. Therefore we use a special script-based parametric mesh generator [4] using as backend the single problem mesh generator Triangle[5] to get the corresponding finite element solution for the magnetic vector potential \( A \). Then from \( A \), we compute the magnetic field density \( B \). From \( B \) we evaluate the objective function \( F \). The method of optimization used will dictate how the parameter set of device description \( \vec{h} \) is to be changed depending on the computed \( F \). In our work we used GA [6] to change the design parameter \( \vec{h} \).

The design parameter vector \( \vec{h} \) is represented by a binary encoding method. A chromosome is a vector \( \vec{h} \). The fitness score \( f \) is defined in terms of the objective function \( F \). Though our objective function is to be minimized, the fitness score has to be maximized. We defined the fitness score \( f = \frac{1}{1+F} \). Therefore when \( F \) goes to 0, \( f \) will reach its maximum 1. The methodology of optimization using GA is shown Fig. 3. First we randomly generate hundreds of vectors \( \vec{h} \) (chromosomes) and this set is termed the initial population. Then fitness score for each \( \vec{h} \) is calculated and checked as to whether it is maximum or not. If not, then the design parameters are changed according to the classical GA way, applying selection, crossover, and mutation [6].
Numerical Model

The numerical example in Fig. 4 is used to validate the proposed algorithm. The coil (with $\mu_r = 1.0$, and current density $J = \pm 5 \times 10^4 \text{ A/m}^2$) excites the magnetic field in the steel plate (with $\mu_r = 100.0$ and current density $J = 0.0$). The conductor is surrounded by air (with $\mu_r = 1.0$ and current density $J = 0.0$). The magnetic field density in the $\hat{y}$ direction $B_y$ is measured at $y = 4.5 \text{ cm}$, $8 \text{ cm} \leq x \leq 12 \text{ cm}$ using 10 points in the interval as shown in Fig. 4 labeled as measuring line.

For our forward problem we set the design parameter $\vec{h} = \{2.0, 1.2, 2.4, 2.2, 2.7, 3.0, 3.2, 3.5\} \text{ cm}$ and find $B_y$ at the measuring points and these are our calculated $B$ values. Each design variable is represented by 10 bits. On each node on the defect, the vertical displacements are selected as design parameters. In our numerical model we have 8 geometric parameters contained in the vector $\vec{h}$ ($h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8$). The measuring line located at $y = 4.5 \text{ cm}$, is sampled into 10 equally spaced points and tolerance boundaries are set to $0.5 \text{ cm} \leq h \leq 3.5 \text{ cm}$.
Results

For testing we took a particular defect $\vec{h}$ and computed the field. We took this to be from measurements. Now our algorithm has to reconstruct $\vec{h}$ to match the “measurements”. Fig. 5. shows the optimum shape of the defect after 200 iterations for a population size 200. The fitness score increased up to 0.99837 for the optimum shape of the defect. This shows an almost perfect reconstruction of the defect. The equipotential lines for the finite element solution magnetic vector potential are shown in Fig. 6.

Fig. 5. Optimum shape of the Reconstructed Defect

Fig. 6. Solutions in Equipotential lines for the Numerical Model
Conclusions

This paper presents a finite element technique for solving inverse problems in magnetostatic NDE. Defect shape reconstructing using the genetic algorithm optimization method is presented and validated using a simple geometry. This problem also will be computed using GPU parallel computing technique and it is observed that it is very much faster than computing without GPU. This model now has to be extended to 3-D.

References


