Incorporating Excel Data Solver in Teaching Undergraduate Engineering Curriculum

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Introduction

For a typical engineering curriculum, occasionally one would run into nonlinear equations that do not have a closed-form solution. This can happen in any engineering mechanics courses when dealing with statically-indeterminate problems or when over-constrained conditions exist. Traditionally these types of problems are only solvable using numerical analysis techniques. But for most undergraduate engineering programs, numerical analysis is a math elective course at best and most students don’t have the time and space in their curriculum to take the course. How do we teach students to properly use a tool (software/program) to solve complex engineering problems without getting into too much numerical analysis jargon? With the wide availability of Microsoft Office Suite, this problem can easily be resolved. Excel has an add-on called “Data Solver” which is easy to learn and use. In most cases it gives satisfactory results if some guidelines are carefully followed. A major advantage of using Excel data solver is, it is free. There is no additional cost incurred if one already has the MS Office Suite. The Excel data solver can be installed/activated as an add-in if it was never used before (see end of this paper for instructions on how to install/activate this add-in).

Having worked in the automotive industry for many years, the author knows full well how important Excel skills are to practicing engineers. Corporations large and small are sending their engineers to get trained in using Excel tools for engineering analysis purposes. Some engineers develop VBA applications on Excel platform so other engineers or analysts can use it. Almost eight years into teaching a wide range of general engineering and mechanical engineering courses at the undergraduate level, the author has yet to find one engineering textbook that uses Excel data solver to tackle nonlinear equations or problems, except books that specifically teaches on how to use Excel15. The author believes this to be an oversight on the part of engineering educators. This paper attempts to address this issue by showing a few case studies on how Excel data solver can be incorporated in a typical undergraduate engineering curriculum solving a wide variety of problems. Examples drawn from engineering mechanics, kinematics and machine design, and engineering economy are used to demonstrate the usefulness of this tool and things to watch for when teaching it.
State-of-the-Art

Some academic endeavor had been taken in recent years to develop Excel-based tools for specific mechanical engineering curriculum. A research team with NSF funding is developing a set of computational and organizational tools to be used in the thermodynamic and heat transfer courses taught in the University of Alabama's Mechanical Engineering department. As the team claims, “The platform for these tools will be the Microsoft Office Suite, mainly Microsoft Excel, due to its practicality and wide usage both in the academic and professional spheres.”1, 2

There are also companies that specialize in developing Excel-based tools or provide training for corporate clients. California-based SpreadsheetWorld, Inc. is such a company that promotes Spreadsheet Aided Engineering3. Many engineering programs teach Excel in freshmen-level introduction to engineering courses. But Excel data solver had not been mentioned or emphasized due to limited hours4. Amir Karimi did give some examples using Excel solver in an undergraduate heat transfer course5.

A Statically-indeterminate Problem

Here is a statics problem at hand:

_A vertical force _F_ is applied to a spring assembly at point _A_ (Fig. 1). The spring constants _k_1 and _k_2 are 500 lb/ft. Determine the force in each spring and the deflection of node _A_ if the magnitude of force _F_ is 500 lb._

![Fig. 1 Two spring assembly before load _F_ is applied](image1)

This problem was taken from an example in Sridhar S. Condoor’s _Engineering Statics_6. Here is a brief summary of the solution. In the free-body diagram of node _A_ (Fig. 2),

![Fig. 2 Free-body diagram of node _A_ after load _F_ is applied (point _A_ now moves down to point _A’_)](image2)
F is given, $F_1$ and $F_2$ are unknowns, and angles $\theta_1$ and $\theta_2$ are also unknowns. Applying static equilibrium equations in both the horizontal and vertical directions, we have

$$F_1 \cos \theta_1 = F_2 \cos \theta_2$$  \hspace{1cm} (1)

$$F_1 \sin \theta_1 + F_2 \sin \theta_2 = 500 \text{ lb}$$  \hspace{1cm} (2)

For four unknowns we have only two equations from (1) and (2). So this is a statically indeterminate problem and geometrical constraints need to be used to solve it. From symmetry we know that $\theta_1 = \theta_2$ and $F_1 = F_2$. If the amount of vertical stretch at point A is $\delta$, and $\delta_1$ and $\delta_2$ are deflections of the springs, as shown in Fig. 3, we then have

$$\delta_1 = \left( \sqrt{4^2 + (3 + \delta)^2} - \sqrt{4^2 + 3^2} \right)$$  \hspace{1cm} (3)

![Fig. 3 Geometry of the spring assembly before and after stretch](image)

Again from symmetry we know $\delta_1 = \delta_2$. From Hooke’s law, we have

$$F_1 = k_1 \delta_1, \quad F_2 = k_2 \delta_2$$  \hspace{1cm} (4)

From Equation (2) we have,

$$2F_1 \sin \theta_1 = 500$$  \hspace{1cm} (5)

Substituting the values of $F_1$ and $\theta_1$, we have

$$2k_1 \delta_1 \sin \theta_1 = 2k_1 \left( \sqrt{4^2 + (3 + \delta)^2} - \sqrt{4^2 + 3^2} \right) \frac{(3+\delta)}{\sqrt{4^2 + (3+\delta)^2}} = 500$$  \hspace{1cm} (6)

In Equation (6), only $\delta$ is unknown. This is clearly a nonlinear equation as $\delta$ is buried in several different radicals, so there are no closed-form solutions. Numerical analysis techniques have to
be used. Re-arranging Equation (6), we construct R as a residue that need to be minimized in the Solver,

\[
R = \left( \sqrt{4^2 + (3 + \delta)^2} - \sqrt{4^2 + 3^2} \right) \frac{(3+\delta)}{4^2+(3+\delta)^2} - 0.5 = 0
\]  

Using Excel data solver, we then enter the residue function R in Equation (7) as an objective function in the parameter input window as shown in Fig. 4, and select delta (\( \delta \)) as the variable cell. Excel would then iteratively solve for delta that would give us a zero value of R.

![Fig. 4 Excel Data Solver Parameter Input Screen](image)

Using an initial guess of delta being equal to 0, we got an answer of 1.063089 ft. Now this problem is easy since it has only one equation as the objective function. So the focus in teaching is to show the students how to properly set up the equations, which variable to select as changing variable and how to properly set up an objective function.

For homework, students were assigned a three spring problem – all the dimensions in Fig. 1 remain the same, just add an identical spring in the middle. Another possible assignment is changing the spring rate so that the two springs are no longer the same. These will all add a little complexity to the modeling and the setup of Excel solver parameters. And in some cases an improper initial guess would give students wrong answers. So students are asked to be vigilant on detecting erroneous answers and learn how to fix them. One way of fixing it is by adding some constraints based on the geometry in the Solver parameter window. Sometimes students would set up the objective function and the changing variables in such a way that circular argument scenario arises. Therefore students need to be taught on making sure all the given information in the problem are used. In a way, even though Excel data solver makes computation easier, but the thinking behind it is sometimes quite challenging and students need to be exposed
to a variety of problems to hone their numerical analysis skills. In the next section, a kinematic design problem is used to demonstrate how to combine multiple equations into a single objective function in Excel solver.

An Example in Kinematics Analysis and Machine Design

Here is an example problem from kinematics analysis. All dimensions were measured off of Example 4-7 of a kinematics and design of machinery textbook.

*Calculate the angular position, velocity and acceleration of each moving link (2, 3, 4, and 5) of the six-link mechanism in its current position (θ₁ = 36°) as shown in Fig. 5.*

θ₁, θ₂, θ₃, θ₄, and θ₅ are defined as shown. All link dimensions are known to be:

\[ L_1 = 10 \text{mm}, \ L_2 = 35.5 \text{mm}, \ L_3 = 13.5 \text{mm}, \ L_4 = 15.5 \text{mm}, \ L_5 = 16 \text{mm}, \ L_6 = 36 \text{mm}, \ X_G = 27 \text{mm}, \ Y_G = 33.5 \text{mm}. \]

DCE is one piece, i.e., angles between DC, CE, and ED do not change. The fixed angle between DE and CD is \( \alpha = 134° \). Assume link 1 is the driving link and it is rotating at a constant angular velocity of \( \omega_1 = 1 \text{ rad/sec} \).

Vector loop equations for the two closed loops ABCDGA and GDEFG are:

\[
\begin{align*}
\ell_1 \cos \theta_1 + \ell_2 \cos \theta_2 + \ell_3' \cos (\alpha + \theta_3) &= x_G + \ell_4 \cos \theta_4 \\
\ell_1 \sin \theta_1 + \ell_2 \sin \theta_2 + \ell_3' \sin (\alpha + \theta_3) &= y_G + \ell_4 \sin \theta_4 \\
\ell_4 \cos \theta_4 + \ell_3 \cos \theta_3 &= \ell_6 + \ell_5 \cos (\theta_5) \\
\ell_4 \sin \theta_4 + \ell_3 \sin \theta_3 &= \ell_5 \sin (\theta_5)
\end{align*}
\]

Fig. 5 A complex six-bar linkage design
Students were initially required to solve for the angles in the above four equations using Matlab nonlinear solver. But there was one student who did not purchase a Matlab license. So the author encouraged him to try to use Excel data solver to solve it instead. But he only knew how to solve problems that have only one equation as the objective function as shown in Equation (7) above, for example. The student had to be taught how to combine the four equations, Eqn’s (8) through (11), into one objective function. If we use four equations $f_1(S) = 0$, $f_2(S) = 0$, $f_3(S) = 0$, $f_4(S) = 0$ to represent Equations (8) through (11), where $S = [\theta_2, \theta_3, \theta_4, \theta_5]$ is a vector of the unknown angles. Then a single objective function can be written as

$$F = f_1^2 + f_2^2 + f_3^2 + f_4^2$$  \hspace{1cm} (12)

The goal of this problem is to drive objective function $F$ to be zero (or as close to zero as possible numerically). The student was then able to solve it correctly (Fig. 6), even using some constraints.

![Excel solution to the sample kinematics problem](image)

**Fig. 6** Excel solution to the sample kinematics problem

### An Example in Structural Design

In some analysis, the need to find min/max condition exists. An end-of-chapter exercise problem (Problem 4.23) from Beer & Johnston’s *Statics and Mechanics of Materials* book was morphed into a structural design problem below:

*The rig shown in Fig. 7 consists of a 1200 lb (self-weight) horizontal member ABC and a vertical member DBE (self-weight not given) welded together at B. The rig is being used to raise a 3600 lb crate at a distance $x$ from the vertical member DBE. The cable is*
anchored at F as shown. The design goal is to make sure (1) fixed support at E would not fail and (2) the cable ADCF would not fail, for the entire range of x, which is from 1.5 ft to 17.5 ft.

It seems there are two design parameters to consider. One is the maximum bending moment at point E. Once this moment is known, the sizing and material/joint selection for column DBE can be done (a decision can be made later on how deep the steel column should be buried in the concrete dock so point E does not fail due to bending moment). The other parameter we need to keep an eye on is the maximum tension in the cable ADCF. Proper tensioning of the cable can actually help minimize the maximum bending moment experienced by the column at point E. One of the most important steps to solving this design problem is using Excel spreadsheet to find the required tension in cable ADCF when the maximum bending moment at point E is minimized. So this is a min/max problem. And it turned out that Excel data solver is a perfect tool for this application. A free body diagram is drawn in Fig. 8.

![Fig. 7 Design of a Rig and Crate Structure](image)

The detailed solution is given here:

\[ \sum M_E = 0: M_E - T(3.75) + 3600x + 1200(6.5) = 0 \]  
(13)

Simplifying Eqn. (13) we have

\[ M_E = (3.75)T - 3600x - 7800 \]  
(14)

For x=1.5 ft, we have

\[ (M_E)_1 = (3.75)T - 3600(1.5) - 7800 \]  
(15)
For \( x = 17.5 \) ft, we have

\[
(M_E)_2 = (3.75)T - 3600(17.5) - 7800
\]  \( (16) \)

Therefore, the final solution for the problem becomes,

(a) For the smallest max value of \(|M_E|\), we set \((M_E)_1 = -(M_E)_2\) and obtain \(T = 11.20\) kips.
(b) Plug the value of \(T\) into Equation \((15)\), we obtain \(|M_E| = 28.8\; \text{kip} \cdot \text{ft}\)
Using Excel, the solution we got using a min/max objective function matches the hand calculation above. Note in this solver calculation, tension in cable (T) is the only changing variable. Since support moment at point E generated by the ground when crate is at \( x_1 = 1.5 \) ft or \( x_2 = 17.5 \) ft are different, and the design goal is to minimize the maximum magnitude of this moment around point E, the only reasonable solution would be to have the moment being equal in magnitude for both positions \( x_1 \) and \( x_2 \). The objective function is \( \max(\text{abs}(M_{E1}), \text{abs}(M_{E2})) \), as shown in Fig. 9.

Another question that could be asked of students is, would it be a good idea to detach the cable from anchor at F and attach it instead to column DBE at a point, say, 1 feet above the concrete floor? It can be left as a homework problem for students to explain their reasoning and show its impact on max bending moment at point E (foot of the column DBE) and tension in the cable.

**An Example in Engineering Economy**

In engineering economy, one of the tasks that one often encounter is to find a loan’s interest rate knowing the amount borrowed, the annual payment, and the number of years of payment. This sounds easy but actually is a nonlinear problem and there is no closed-form solution. Traditional engineering economy textbooks would spend pages deriving solutions using a combination of graphical and interpolation techniques in order to get an approximate solution. With Excel data solver, the whole trouble of interpolation can be avoided. Now we will show one example problem (Example 3.9 from Ted. G. Eschenbach’s book, *Engineering Economy: Applying Theory to Practice*).

*If $10,000 is borrowed and payments of $2,000 are made each year for 9 years, what is the interest rate?*

Knowing present worth \( P = $10,000 \), and annuity \( A = $2,000 \), and periods \( N = 9 \), we need to solve for interest rate \( i \). Since \( $10,000 = (A/P, i, N) = $2,000 \), and from the formula for \( (A/P, i, N) \):

\[
(A/P, i, N) = \frac{1}{(P/A, i, N)} = \frac{i((1+i)^N)}{(1+i)^N - 1} \tag{17}
\]

Excel solver input window is shown in Fig. 10. Note here interest rate is the changing variable and the objective function is a single equation as shown in the formula above.

**Limitations of the Excel Data Solver**

It has been documented that Excel Data Solver has an upper limit of 200 for decision variables, and limits on the number of constraints that can be used. To increase those limits, premium has to be paid to purchase Excel add-ons supplied by 3rd-party vendors. At least one vendor is claiming an upper limit of 8,000 decision variables.

Another limitation of Excel Data Solver is its speed during optimization, especially when the number of independent variables become large. Commercial Excel add-ons such as the ones supplied by *Optimum Power Technology, Inc.* in Pittsburgh, PA, had demonstrated some success in solving this speed problem using advanced design of experiment techniques. There are three...
patented model-based design optimization software products in *Optimum Power Technology’s* newest line of software. *Global-OPT* is free and Multi-Objective-OPT are Excel Add-Ins that optimize and calibrate Excel and MATLAB models. The *Design Optimizing Expert System (DOES)* can optimize simulation models created with Computer Aided Engineering (CAE) tools such as ANSYS and FLUENT\(^\text{12}\).

![Fig. 10 Excel solution](image)

But for most engineering problems students encounter in their undergraduate education, the standard Excel data solver proves to be more than enough for use. Note that Mac and PC have slightly different instructions on how to activate the Excel data solver add-in. and detailed instructions for both platforms can easily be found online. For example, if you use a Mac computer, here are a few the steps on how to install the free Solver add-in\(^\text{13}\):

1. Start Excel for Mac.
2. Click *Tools*, and then click *Add-Ins*.
3. Click the *Data Analysis ToolPak* or *Solver* option to enable it. Then, click OK.
4. Locate *Data Analysis ToolPak* or *Solver* on the *Data* tab.

For instructions on installing Solver in Windows PC, see reference\(^\text{14}\).

**Summary**

This paper attempts to show that Excel data solver is an excellent tool to teach students to use and tackle nonlinear problems encountered in engineering analysis and design. Excel data solver can be easily incorporated in a typical undergraduate engineering curriculum either as part of the introduction to engineering class or as part of a lab or as part of any engineering curriculum. It can solve a wide variety of problems. Examples drawn from engineering mechanics, kinematics
and machine design, and engineering economy are used to demonstrate the usefulness of this technique and things to watch for when teaching it.

Bibliography

1. [http://www.me.ua.edu/ExcelinME](http://www.me.ua.edu/ExcelinME), accessed on 1/29/2016

Biographical Information

David Che had taught at an ABET accredited engineering program at Geneva College in Beaver Falls, PA for seven years before joining Anderson University, Anderson, Indiana in 2015. He received his B.S.E in precision mechanical engineering from Harbin Institute of Technology, Harbin, P. R. China; M.S. in mechanical engineering from Ohio State University, Columbus, Ohio; and Ph.D in mechanical engineering from University of Michigan, Ann Arbor, Michigan. He was a senior research/project engineer at General Motors Corporation in Warren, Michigan from 1997-2005. He also served as General Manager of Stafast Products, Inc. ’s Asia operations in Shenzhen and Dongguan of China from 2005-2008. He will be joining a new engineering program at Mount Vernon Nazarene University in Mount Vernon, Ohio this fall.