Polar Equation Exercises

The identities \( \cos^2 \theta = \frac{1}{2} + \frac{1}{2} \cos 2\theta \) and \( \sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos 2\theta \) may be useful for some integration in these exercises.

1. Figure 1 shows a portion of the graph of the polar equation, \( r = \theta \).
   (a) Find an equation (rectangular form) for the tangent line to \( r = \theta \) at the point where \( \theta = \frac{\pi}{4} \).
   (b) Find the exact value of the shaded area.

   ![Figure 1](image1.png)

2. Figure 2 shows the graph of the polar equation, \( r = 2 + \sin \theta \). Find the exact value of the shaded area.

3. Figure 3 shows the graph of the polar equation, \( r = 1 + \cos \theta \).
   (a) Find an equation (rectangular form) for the tangent line to \( r = 1 + \cos \theta \) at the points where \( \theta = \pm \frac{\pi}{2} \).
   (b) Find the points (rectangular coordinates) on the graph of \( r = 1 + \cos \theta \) where the tangent line is horizontal and the points where the tangent line is vertical.
   (c) What information about the area of the enclosed region is available from the result of part (b)? Find the exact value of the enclosed area.

   ![Figure 2](image2.png)

   ![Figure 3](image3.png)
4. Figure 4 shows the graphs of the polar equations, $r = 1 + \cos \theta$ and $r = \cos \theta$. Find the exact value of the shaded area.

5. Figure 5 shows the graphs of the polar equations, $r = 1 + \cos \theta$ and $r = 1 + \sin \theta$. Find the exact value of the shaded area.