Kirchhoff's Laws

A. Kirchhoff's Loop Law

Suppose that a charged particle moves as shown below from point A to point B, then from point B to point C, and then from point C back to point A. Its potential energy will not have changed since it is back where it started.

Kirchhoff's loop law is an application of this idea: The sum of voltage changes around a closed loop is zero.

Symbolically, the potential changes for the path described are:

\[(V_B - V_A) + (V_C - V_B) + (V_A - V_C) = 0\]

As an example, consider a circuit consisting of a voltage source \(V\) and two resistors \(R_1\) and \(R_2\) in series.

![Circuit Diagram]

Start from the negative side of the voltage source and traverse the circuit clockwise.

1. voltage gain \(V_B - V_A = V\) (voltage of the source)
2. voltage loss \(V_C - V_B = -IR_1\) (voltage drop across \(R_1\))
3. voltage loss \(V_A - V_C = -IR_2\) (voltage drop across \(R_2\))

Kirchhoff's loop law states that

\[V - IR_1 - IR_2 = 0\]
Then

\[ V = IR_1 + IR_2 \]
\[ V = I(R_1 + R_2) \]

The equivalent resistance of \( R_1 \) and \( R_2 \) in series is

\[ \frac{V}{I} = R_1 + R_2 = R_s \]

In general, if there are \( n \) resistors in series, then the equivalent resistance is given by

\[ R_s = R_1 + R_2 + R_3 + \ldots + R_n \]

B. Kirchhoff’s Point Law

The conservation of electric charge, when applied to circuits, gives Kirchhoff’s point of law: **The net current flowing into a junction equals the net current flowing out.**

As an example, consider a circuit consisting of a voltage source connected to two resistors in parallel.

Kirchhoff’s current point states that:

\[ I = I_1 + I_2 \]

Kirchhoff’s loop law applied to the two loops above states that

\[ V = I_1R_1 \] and \[ V = I_2R_2 \]

The equivalent resistance of \( R_1 \) and \( R_2 \) in parallel is

\[ \frac{V}{I} = \frac{V}{I_1 + I_2} = \frac{1}{1/R_1 + 1/R_2} \]
\[ = \frac{R_1R_2}{R_1 + R_2} = R_p \]

In general, if there are \( n \) resistors in parallel, then the equivalent resistance is given by

\[ \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \]
Procedure - Experiment 2

A. Resistors in Series

1. Measure the values of the three resistors using the digital ohmmeter.

2. Connect the three resistors in series, as shown.

![Diagram of resistors in series](image)

Measure the resistance of this series combination with the digital ohmmeter. Compare this measurement with the value computed from equivalent resistance equation for resistors in series.

3. Set up the circuit shown below and measure the current $I$. The power supply should be set to 12 volts.

![Diagram of circuit with power supply](image)

Compute the resistance of the series combination of resistors from $V/I$. 
4. Remove the digital milliammeter from the circuit and switch it to an appropriate voltage range. Measure the voltages $V_1$, $V_2$, $V_3$ across the three resistors. For example, the diagram below shows the measurement of $V_2$.

![Image of voltage measurement setup]

Verify Kirchhoff's loop law: $V_1 + V_2 + V_3 = 12$ volts.

5. Compute the voltage drops across the three resistors by using the voltage divider equations:

$$V_1 = 12 V \left(\frac{R_1}{R_s}\right) \quad V_2 = 12V \left(\frac{R_2}{R_s}\right) \quad V_3 = 12V \left(\frac{R_3}{R_s}\right)$$

B. 1. Connect the three resistors in parallel, as shown.

![Image of parallel resistor setup]

Measure the resistance of this parallel combination with the digital ohmmeter. Compare this measurement with the value computed from the equivalent resistance equation for parallel resistors.
2. Set up the circuit shown below and measure the total current $I$.

Compute the resistance of the parallel combination of resistors from $V/I$.

3. Put the digital milliammeter in each branch of the parallel resistor combination and measure the currents $I_1$, $I_2$, $I_3$. Verify Kirchhoff's current law:

$$I_1 + I_2 + I_3 = I$$

4. Compute the three currents by using the current divider equations:

$$I_1 = I \left( \frac{R_p}{R_1} \right) \quad I_2 = I \left( \frac{R_p}{R_2} \right) \quad I_3 = I \left( \frac{R_p}{R_3} \right)$$

where $R_p$ is the equivalent resistance of the parallel combination.

C. Resistors in a Series and Parallel Combination

1. Connect the three resistors as shown below.

Measure the resistance of this combination with the digital ohmmeter. The equivalent resistance of this combination is given by:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{(R_2+R_3)}$$

Use this equation to compute $R_{eq}$ and compare with the measured value.
2. Set up the circuit shown below.

![Diagram of the circuit](image)

Compute the current delivered by the battery using \( I = \frac{V}{R_{eq}} \).

Using the digital milliammeter measure \( I, I_1, I_2, \) and \( I_3 \). Switch the meter to an appropriate voltage range and measure \( V_1, V_2, \) and \( V_3 \).

3. Connect the three resistors as shown below.

![Diagram of the resistors](image)

Measure the resistance of this combination with the digital ohmmeter. The equivalent resistance of this combination is given by:

\[
R_{eq} = \frac{R_1 R_2}{R_1 + R_2} + R_3
\]

Use this equation to compute \( R_{eq} \) and compare with the measured value.
4. Set up the circuit shown below.

Compute the current delivered by the battery using \( I = \frac{V}{R_{\text{eq}}} \).

Using the digital milliammeter measure \( I, I_1, I_2, \) and \( I_3 \). Switch the meter to an appropriate voltage range and measure \( V_1, V_2, \) and \( V_3 \).
A. Resistors in Series

1. Measured values of the three resistors
   \[ R_1 = \_\_\_\_\_\_\_\_ k\Omega \quad R_2 = \_\_\_\_\_\_\_\_ k\Omega \quad R_3 = \_\_\_\_\_\_\_\_ k\Omega \]

2. Measured value of the series combination
   \[ R_s(\text{measured}) = \_\_\_\_\_\_\_\_ k\Omega \]

3. Computed value of the series combination using V/I
   \[ I = \_\_\_\_\_\_\_\_ mA. \quad (12 \ V.)/I = \_\_\_\_\_\_\_\_ k\Omega \]

4. Measured values of \( V_1, V_2, V_3 \)
   \[ V_1 = \_\_\_\_\_\_\_\_ volts \quad V_2 = \_\_\_\_\_\_\_\_ volts \quad V_3 = \_\_\_\_\_\_\_\_ volts \]
   \[ V_1 + V_2 + V_3 = \_\_\_\_\_\_\_\_ volts \]

5. Computed values of \( V_1, V_2, V_3 \)
   \[ V_1 = \_\_\_\_\_\_\_\_ volts \quad V_2 = \_\_\_\_\_\_\_\_ volts \quad V_3 = \_\_\_\_\_\_\_\_ volts \]

B. Resistors in Parallel

1. Measured value of the parallel combination
   \[ R_p(\text{measured}) = \_\_\_\_\_\_\_\_ k\Omega \]
   Computed value of the parallel combination
   \[ R_p(\text{computed}) = \_\_\_\_\_\_\_\_ k\Omega \]

2. Computed value of the parallel combination using V/I
   \[ I = \_\_\_\_\_\_\_\_ mA. \quad (12 \ V.)/I = \_\_\_\_\_\_\_\_ k\Omega \]
3. Measured values of $I_1$, $I_2$, $I_3$

\[ I_1 = \ldots \text{mA.} \quad I_2 = \ldots \text{mA.} \quad I_3 = \ldots \text{mA.} \]

\[ I_1 + I_2 + I_3 = \ldots \text{mA.} \]

4. Computed values of $I_1$, $I_2$, $I_3$

\[ I_1 = \ldots \text{mA.} \quad I_2 = \ldots \text{mA.} \quad I_3 = \ldots \text{mA.} \]

C. Resistors in a Series and Parallel Combination

1. Measured value of the combination

\[ R_{eq} \text{ (measured)} = \ldots \text{kΩ} \]

Computed value of the combination

\[ R_{eq} \text{ (computed)} = \ldots \text{kΩ} \]

2. Computed value of the current.

\[ I = \ldots \text{mA} \]

Measured values of $I$, $I_1$, $I_2$, $I_3$

\[ I = \ldots \text{mA} \quad I_1 = \ldots \text{mA} \quad I_2 = \ldots \text{mA} \quad I_3 = \ldots \text{mA} \]

What relations do you see between these currents?
Measured values of $V_1$, $V_2$, $V_3$

\[
V_1 = \underline{\phantom{0}} \text{ Volts} \quad V_2 = \underline{\phantom{0}} \text{ Volts} \quad V_3 = \underline{\phantom{0}} \text{ Volts}
\]

What relations do you see between these voltages?

Would any readings change if $R_1$ and $R_2$ were interchanged?

3. Measured value of the combination

\[
R_{\text{eq}} \ (\text{measured}) = \underline{\phantom{0}} \text{ k}\Omega
\]

Computed value of the combination

\[
R_{\text{eq}} \ (\text{computed}) = \underline{\phantom{0}} \text{ k}\Omega
\]

4. Computed value of the current.

\[
I = \underline{\phantom{0}} \text{ mA}
\]

Measured values of $I$, $I_1$, $I_2$, $I_3$

\[
I = \underline{\phantom{0}} \text{ mA} \quad I_1 = \underline{\phantom{0}} \text{ mA} \quad I_2 = \underline{\phantom{0}} \text{ mA} \quad I_3 = \underline{\phantom{0}} \text{ mA}
\]

What relations do you see between these currents?
Measured values of $V_1$, $V_2$, $V_3$

$V_1 = \underline{\hspace{2cm}}$ Volts $\quad V_2 = \underline{\hspace{2cm}}$ Volts $\quad V_3 = \underline{\hspace{2cm}}$ Volts

What relations do you see between these voltages?

Would any readings change if $R_1$ and $R_2$ were interchanged?