1. (2 points) Use the definition of the limit to prove that \( \lim_{x \to 3} \frac{x}{5} = \frac{3}{5} \).

**Solution:**
Let \( \varepsilon > 0 \) be given. We need to find \( \delta > 0 \) such that \( 0 < |x - 3| < \delta \) implies that

\[
\left| \frac{x}{5} - \frac{3}{5} \right| < \varepsilon.
\]

In order to find such \( \delta \), we solve the latter inequality for \( |x| \) in terms of \( \varepsilon \):

\[
\left| \frac{x}{5} - \frac{3}{5} \right| = \frac{1}{5} |x - 3| < \varepsilon.
\]

Since we have \( \frac{1}{5} |x - 3| < \varepsilon \), this suggests that we should choose \( \delta = 5 \varepsilon \). So, if \( 0 < |x - 3| < 5\varepsilon \), then

\[
\left| \frac{x}{5} - \frac{3}{5} \right| = \frac{1}{5} |x - 3| < \frac{1}{5} \cdot 5\varepsilon = \varepsilon
\]

as needed.

2. (2 points) Use the definition of the limit to prove that \( \lim_{x \to -3} \frac{1}{(x+3)^4} = \infty \).

**Solution:**
Let \( M > 0 \) be given. We need to find a \( \delta \) such that \( 0 < |x - (-3)| = |x + 3| < \delta \) implies that

\[
\frac{1}{(x+3)^4} > M.
\]

So, to find such \( \delta \), we solve the latter inequality for \( x + 3 \) in terms of \( M \):

\[
\frac{1}{(x+3)^4} > M
\]

\[
(x+3)^4 < \frac{1}{M}
\]

\[
|x + 3| < \frac{1}{\sqrt[4]{M}}.
\]

So choose \( \delta = 1/\sqrt[4]{M} \). Then

\[
\frac{1}{(x+3)^4} > \frac{1}{(1/\sqrt[4]{M})^4} = \frac{1}{1/M} = M
\]

as needed.
3. (2 points) Show that there is a solution to \( \ln x = e^{-x} \) on the interval \((1, 2)\).

Solution:
We need to use the intermediate value theorem to find a root of the equation \( \ln x - e^{-x} = 0 \). Using the terminology in the statement of the theorem, we have \( f(x) = \ln x - e^{-x}, (a, b) = (1, 2) \) and \( N = 0 \). So we need to first find \( f(1) \) and \( f(2) \). Using a calculator, we see \( f(1) \approx -0.3679 \) and \( f(2) \approx 0.5578 \). So 0 is between \( f(1) \) and \( f(2) \). Therefore, by the theorem, there exists a \( c \in (1, 2) \) such that \( f(c) = 0 \). Thus \( \ln c = e^{-c} \) as required.

4. (2 points) The displacement (in meters) of a particle moving in a straight line is given by the equation of motion \( s(t) = \frac{1}{t^2} \), where \( t \) is measured in seconds. Find the velocity of the particle at times \( t = a \), \( t = 1 \), \( t = 2 \) and \( t = 3 \).

Solution:
We need to use the definition of the derivative to find the derivative of \( s(t) \) at \( t = a \). We have

\[
\lim_{h \to 0} \frac{1/(t + h)^2 - 1/t^2}{h} = \frac{1}{t^2 + 2th + h^2} - \frac{1}{t^2} = \lim_{h \to 0} \frac{t^2 - t^2 - 2th - h^2}{h(t^4 + 2t^3h + t^2h^2)} = \lim_{h \to 0} \frac{-2th - h^2}{h(t^4 + 2t^3h + t^2h^2)} = \lim_{h \to 0} \frac{-2t - h}{h(t^4 + 2t^3h + t^2h^2)} = \lim_{h \to 0} \frac{-2t}{t^4 + 2t^3h + t^2h^2} = \frac{-2}{t^3}.
\]

So the derivative of \( s(t) \) at \( t = a \) is \(-2/t^3\). Using this formula, we plug in 1, 2 and 3 to get the corresponding velocities: \( s'(1) = -2 \), \( s'(2) = -1/4 \) and \( s'(3) = -2/27 \).

5. (2 points) Find the derivative of \( f(x) = x + \sqrt{x} \) using the definition of the derivative. State the domain of \( f \) and the domain of \( f' \).

Solution:
Using the definition of the derivative, we have

\[
\lim_{h \to 0} \frac{(x + h + \sqrt{x + h}) - (x + \sqrt{x})}{h} = \lim_{h \to 0} \frac{x + h + \sqrt{x + h} - x - \sqrt{x}}{h}
\]

\[
= \lim_{h \to 0} \frac{h + (\sqrt{x + h} - \sqrt{x})}{h}
\]

\[
= \lim_{h \to 0} \frac{h}{h} + \lim_{h \to 0} \frac{\sqrt{x + h} - \sqrt{x}}{h}
\]

\[
= 1 + \lim_{h \to 0} \left[ \frac{\sqrt{x + h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x + h} + \sqrt{x}}{\sqrt{x + h} + \sqrt{x}} \right]
\]

\[
= 1 + \lim_{h \to 0} \frac{x + h - x}{h(\sqrt{x + h} + \sqrt{x})}
\]

\[
= 1 + \lim_{h \to 0} \frac{h}{h(\sqrt{x + h} + \sqrt{x})}
\]

\[
= 1 + \lim_{h \to 0} \frac{1}{\sqrt{x + h} + \sqrt{x}}
\]

\[
= 1 + \frac{1}{2\sqrt{x}}.
\]

Since \( f \) is a polynomial, it’s range is all of \( \mathbb{R} \). Since \( f' \) is a rational function such that its denominator is 0 when \( x = 0 \), \( f' \) has domain \((-\infty, 0)\) and \((0, \infty)\).